equidistant from BC, CD, and BD, and hence equidistant from the planes A-BC, A-CD, and A-BD. Hence O lies on the line of intersection of the three planes bisecting the dihedral angles C-AB-D, B-AD-C, and D-AC-B.

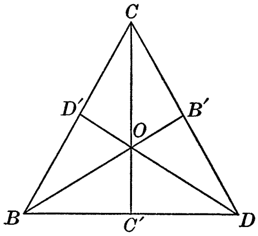

FIG. 1.

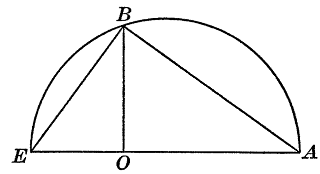

FIG. 2.

But these three planes intersect in the diameter through A of the circumscribed sphere. Extend AO to meet the sphere again in E (Fig. 2). Then EA = diameter of circumsphere $= 2R$. Since A-BCD is a regular tetrahedron, AO is perpendicular to the base, BCD. Hence $\angle AOB$ is a right angle. Also $\angle EBA$, being inscribed in a semicircle, is a right angle. Hence in right $\triangle BOA$,

$$\overline{OA}^2 = \overline{AB}^2 - \overline{BO}^2 = a^2 - \frac{a^2}{3} = \frac{2a^2}{3},\ OA = \frac{a\sqrt{2}}{\sqrt{3}}.$$

$$\text{In right } \triangle EBA,\ \overline{BA}^2 = EA.\quad OA = 2R\cdot\frac{a\sqrt{2}}{\sqrt{3}}.$$

$$a^2 = 2R\cdot\frac{a\sqrt{2}}{\sqrt{3}},\qquad R = \frac{a\sqrt{3}}{2\sqrt{2}} = \frac{a\sqrt{6}}{4}.$$

Also solved by R. M. MATHEWS, NATHAN ALTSHILLER, A. M. HARDING, CLIFFORD N. MILLS, WALTER C. EELLS, A. H. HOLMES, HORACE OLSON, J. C. CLAGETT, J. W. CLAWSON.

CALCULUS.

361. Proposed by EMMA M. GIBSON, Drury College.

Determine the system of curves satisfying the differential equation

$$[(1 + x^2)^{1/2} + ny]dx + [(1 + y^2)^{1/2} + nx]dy = 0,$$

and show that the curve which passes through the point $(0, n)$ contains as part of itself the conic

$$x^2 + y^2 + 2xy(1 + n^2)^{1/2} = n^2.$$

(From Forsyth's *Differential Equations*, p. 41.)

SOLUTION BY GEO. W. HARTWELL, Hamline University.

The terms of the given differential equation may be arranged as follows:

$$(1 + x^2)^{\frac{1}{2}}dx + (1 + y^2)^{\frac{1}{2}}dy + nxdy + nydx = 0 \tag{1}$$

and the equation integrated immediately, giving

$$x\sqrt{1 + x^2} + y\sqrt{1 + y^2} + 2nxy + \log(x + \sqrt{1 + x^2})(y + \sqrt{1 + y^2}) = c. \tag{2}$$

The equation of the curve of this system passing through $(0, n)$ is then

$$x\sqrt{1+x^2}+y\sqrt{1+y^2}-n\sqrt{1+n^2}+2nxy + \log\left[\frac{(x+\sqrt{1+x^2})(y+\sqrt{1+y^2})}{n+\sqrt{1+n^2}}\right]=0. \quad (3)$$

Solving the equation of the conic for x we have

$$x = -y\sqrt{1+n^2} \pm n\sqrt{1+y^2}.$$

These values of x satisfy (3); hence, the conic must be a part of the curve represented by (3).

362. Proposed by C. N. SCHMALL, New York City.

Having given $y^3 - a^2y + axy - x^3 = 0$, show by Maclaurin's theorem that

$$y = -\frac{x^3}{a^2} - \frac{x^4}{a^3} - \frac{x^5}{a^4} - \cdots,$$

SOLUTION BY A. M. HARDING, University of Arkansas.

We obtain by successive differentiation

$$(3y^2 - a^2 + ax)\frac{dy}{dx} - 3x^2 + ay = 0,$$

$$(3y^2 - a^2 + ax)\frac{d^2y}{dx^2} + 6y\left(\frac{dy}{dx}\right)^2 + 2a\frac{dy}{dx} - 6x = 0,$$

$$(3y^2 - a^2 + ax)\frac{d^3y}{dx^3} + 18y\frac{d^2y}{dx^2}\frac{dy}{dx} + 6\left(\frac{dy}{dx}\right)^3 + 3a\frac{d^2y}{dx^2} - 6 = 0,$$

$$(3y^2 - a^2 + ax)\frac{d^4y}{dx^4} + 24y\frac{d^3y}{dx^3}\frac{dy}{dx} + 18y\left(\frac{d^2y}{dx^2}\right)^2 + 36\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^2 + 4a\frac{d^3y}{dx^3} = 0,$$

$$(3y^2 - a^2 + ax)\frac{d^5y}{dx^5} + 30y\frac{d^4y}{dx^4}\frac{dy}{dx} + 60y\frac{d^3y}{dx^3}\frac{d^2y}{dx^2} + 60\frac{d^3y}{dx^3}\left(\frac{dy}{dx}\right)^2 + 90\left(\frac{d^2y}{dx^2}\right)^2\frac{dy}{dx} + 5a\frac{d^4y}{dx^4} = 0.$$

When $x = 0$, $y = 0$, a, or $-a$. Choosing the first value of y, we obtain

$$\frac{dy}{dx} = 0,\quad \frac{d^2y}{dx^2} = 0,\quad \frac{d^3y}{dx^3} = -\frac{6}{a^2},\quad \frac{d^4y}{dx^4} = -\frac{24}{a^3},\quad \frac{d^5y}{dx^5} = -\frac{120}{a^4}.$$

Hence,

$$y = -\frac{6}{a^2}\frac{x^3}{3!} - \frac{24}{a^3}\frac{x^4}{4!} - \frac{120}{a^4}\frac{x^5}{5!} - \cdots$$

$$= -\frac{x^3}{a^2} - \frac{x^4}{a^3} - \frac{x^5}{a^4} - \cdots.$$

Also solved by PAUL CAPRON and GEO. W. HARTWELL.