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THE EXTERNALITY OF RELATIONS.

NO other logical question is more important for contemporary controversy than that of the externality or essentiality of relations; and none is in greater need of clear formulation. That all varieties of opinion in the matter are currently entertained is well known. The neo-Hegelians and their allies hold to the theory of essentiality in its extreme form. The realists, or neo-Leibnizians, as they may be called, hold as firmly to the theory of externality. And the pragmatists occupy the position of common-sense mediators, setting down some relations as essential and some as external; or else holding that any relation may be external or essential according to the purpose of the moment. Yet it may be questioned whether the disagreement between the various parties is as wide as their mutual misunderstanding. To adopt a striking phrase of Ludwig Stein's, "*Sie philosophieren einander vorbei.*"

Under these circumstances what appears to be most needed is not argument but analysis. For the issues are not one but many, and in current controversy they have been almost inextricably entangled. It will be the main object of the present paper to formulate a few of the more important problems that have been confounded. It seems to me that when these problems are properly distinguished, their solution is a comparatively simple matter.

There are two points which I shall have to take for granted, but which, I suppose, will be readily conceded. The first is that there are no entities which we conceive as standing in no relations. In particular, if there are existing things other than our own ideas (in the widest sense of the term), relations are conceived to exist between such things. We do indeed recognize relations between sensations, images, feelings, desires, etc. But when we say, for example, that gold is heavier than iron, the terms of the relation are understood to be gold and iron, whether

gold and iron are identical with our concepts of them or not. Accordingly, I shall assume it to be an error to hold that the terms between which relations are conceived to obtain are mere ideas, as distinguished from the real things to which ideas refer. This assumption is tolerably safe, because no modern thinker of importance (except perhaps Locke) would have thought of questioning it.

In the second place I shall assume that propositions of the form ArB (or, A stands in a certain relation to B) may be as clear to us as any other propositions whatsoever. This does not mean that our concepts of relations are absolutely clear and distinct and final; but simply that no other class of concepts is to be regarded as distinctly superior to them. We have no reason to suppose, for example, that 'gray' is clearer than 'darker than'; that 'line' is clearer than 'between'; or that 'class' or 'proposition' is clearer than 'includes' or 'implies.' This assumption is also, I think, quite safe, though in former times many thinkers would have objected to it. For recent logical studies have proved that concepts of relations must always have a place among the fundamental assumptions of every department of thought. Every set of geometrical axioms, for example, must contain some indefinable relation such as 'collinear with,' 'between,' or 'farther apart than,' as well as some indefinable entity such as 'point.'

The doctrine of the externality of relations appears in three principal forms, which we shall consider in order.

In its first form the doctrine is to the effect that relations are external to the essential nature of all realities, whether these realities are conceived as individuals, as classes, or as ideal types. What is, is; and it is what it is, without consideration of anything else. By 'reality' is in general meant an object of possible knowledge, where knowledge is supposed to be distinguished from opinion by its absolute certitude. In this form, the doctrine is doubtless as old as the distinction between the essential and the non-essential; but it finds its first clear expression in the philosophy of Plato. Consider, for example, the definition of justice in the *Republic*. Is it not extraordinary that justice in the

individual (or in the state) should be defined in terms which take no account of the individual's relations to other individuals (or of the state's relations to other states)? But Plato's thought is, of course, that the just man (or state) must have a certain inner nature of its own, which underlies and accounts for its characteristic behavior in its various relations. This inner nature, then, is what a satisfactory definition of justice must set forth. Why is it, on the other hand, that the thoughtful mind cannot rest content with sensible things as true realities? It is because all that we can say of them is relative. *A* is great as compared with *B*, and small as compared with *C*. Attempt to treat it as a substance which is great or small in itself, and you make it both great and small, and thus fall into irretrievable contradiction.

The doctrine thus takes for granted that realities have essences, which are either simple, and thus undefinable, or are definable in simple terms; and it affirms that the relations in which a substance stands form no part of its essence. It is obvious that a similar doctrine may apply to certain of the *qualities* of things; that is to say, these may be divided into essential qualities, or attributes, and non-essential, or external, qualities. It thus appears that the question of the externality or essentiality of relations is logically subsidiary to the question, whether, and how far, the distinction between the essential and the external is valid; that is to say, whether, and within what limits, adequate definition—definition in terms of the absolutely simple—is possible. For it is on such definition that the distinction between the essential and the non-essential, in its strictest acceptance, rests.

In order to avoid the discussion of this deeper question, let us for the moment confine our attention to the field in which, if anywhere, adequate definition is possible—namely, the mathematical sciences; and let us assume that the definitions of mathematics are, or may be made, entirely adequate. If, then, the externality of relations can be demonstrated here, it becomes highly probable as a general theory. If it is false here, it loses all claim to our allegiance.

Now it is precisely in the field of mathematics that the theory of

externality is most evidently false. For if we consider the indefinables of any branch of mathematics, and ask how their meaning is expressed or conveyed—how, for example, one geometrician can be sure that he means the same thing by ‘collinear’ as other geometricians do—it is at once obvious that it is only by means of the set of axioms in which they appear. The indefinables may, to be sure, have for each man an additional common-sense meaning, which may vary somewhat from man to man; but this is rigidly excluded from scientific discussion. *For the science* the indefinables mean what the axioms make them mean. Their relations to each other, as set forth by the axioms, *are* their meaning, so far as mathematics is concerned. If this is true of the indefinables, it is true *a fortiori* of all other mathematical terms—that their meaning is constituted by their relations; not to mention the fact, that it is always (to an unknown extent) indeterminate what terms ought to be chosen as the indefinables in which to define the rest.

Let these statements not be misunderstood. It is always necessary in the applications of mathematics that there be some means of recognition by which we may be assured that the same classes of phenomena are constantly used to exemplify the same indefinable terms—that ‘point’ in one axiom is given the same denotation as ‘point’ in the other axioms. But what the mark of recognition may be, matters not at all. Thus a logic of classes has been devised, all of whose axioms apply equally well to the classes within a given universe and to the areas within a given total area. For the purposes of this sort of logic an area *is* a class—except that one must not mix together in the same discourse areas and the classes ordinarily so called. It may, then, be said that the meaning of the mathematical indefinables, as expressed in the axioms, is strictly external to the additional meaning which the terms invariably possess in the ‘concrete’ applications of the science. But that would hardly warrant us in saying that the scientific meaning is external to the essence of the terms in question. If anything is ‘external,’ or unessential, it is that additional particularity of meaning which the application involves.

Judging, then, by the example of the mathematical sciences,

we are led to reject decisively the classical doctrine of the externality of relations; and now it may be observed that very similar considerations apply in all other realms of thought. In order that conceptions may be definitely communicable, they must be reduced to conceptions of relations. The very meaning of words is determined by usage—by the way in which the words are connected with each other and with concomitant circumstances. There is no way of directly comparing your conception of red with mine. Their likeness, when critically examined, means no more than this; that they are similarly related to other conceptions which are accepted as alike. This is true even of conceptions of relations. These can be definitely expressed only by means of relations of relations. In the last resort, of course, the whole possibility of communication rests upon the fact that men feel somewhat the same under similar circumstances. Thus we take for granted, until the contrary is proved, that what is red for one man is red for another, and that each man's red differs from gray and green. This assumed, or rather *presumed*, likeness of our experiences in company with one another forms the point of departure for all science. It is, however, a point of departure that has constantly to be criticised and corrected; and each more definite formulation takes the form of a more accurate statement of the relations in which the term in question stands. Thus, while it is true that the objects of our experience are never wholly analyzable into relations—that our world is not a system of relations of relations in which nothing is related—nevertheless it remains true that the clearer and clearer our conceptions of the world become—the more closely they approach the mathematical type—the more largely they may be expressed in relational terms. Whether there is any final limit to this process in brute data of feeling in which no relational content is to be detected may be worth discussing, but it does not concern us here.

The question remains, whether, while some relations are essential, others may not be external, or non-essential. Here again the ulterior question is, what validity the distinction between essential and external possesses. For if nothing is external to anything else, assuredly no relations are external. But here also

the ulterior question may be safely shelved. For if the distinction between essential and unessential is admitted to have any application at all, then surely some relations of some things are unessential to them. No definition can include everything. Generally speaking, the progress of our knowledge is marked by a deepening as well as clarifying of our conceptions. They mean more, as well as mean more definitely. Thus both qualities and relations which have formerly been conceived as external become included in the essence; and there is no definitely assignable limit to this process. Even temporary qualities and relations, which at first view appear to be clearly superficial, since the reality persists when they have passed away, may become essential when they are seen to characterize a definite stage in a typical order of development. In this sense the puppy's blindness and dependence upon his mother are essential to the hound. It may be added, though this is a little aside from the question, that the capacity for entering into temporary relations and (under the requisite conditions) of exhibiting the temporary qualities is often clearly essential. It does not belong to the essence of water to be liquid or solid. But to freeze at a certain temperature, and melt again as it grows warmer, is essential to it if anything is. Or, to take Descartes's famous example of the piece of wax, it is from our present point of view ridiculous to say that what the wax really *is* is simply a certain mode of extended substance. The behavior of the wax under varying conditions (which is, of course, conceivable as its relations to these conditions) is what essentially characterizes it. What in all respects behaves like wax *is* wax.

So much for the first and historically most important form of the doctrine of externality. Its influence pervades the whole of ancient and modern rationalism, and has not been without effect upon empirical movements also. That what a real entity is in itself is one thing and its transitory and even permanent relations are another, is a delusion if ever there was one. Like many other traditional delusions it reaches its climax in the strangely contrasted philosophies of Leibniz and Spinoza. Spinoza sets it down at the forefront of his system. Substance is that a con-

ception of which can be formed independently of every other conception. And Leibniz's windowless monads, while ideally connected in a universal harmony, are none the less each absolutely independent of its relations to the rest. Few of us are now inclined to worship Hegel; but one debt to him we must not forget. It is he who said: "The proposition that we cannot know the nature of things in themselves has passed for an important piece of wisdom. Things are in themselves in so far as abstraction is made from all that they are for other things—which is as much as to say, in so far as they are thought of as without any characteristics at all, as mere nothings. In this sense it is true enough that one cannot know *what* the thing in itself is."

We must now note in passing a second form of the doctrine, in which the notion of essentiality has fallen away. The question now is, whether the relations in which a thing stands are external to its qualities, essential or external. Can a thing enter into a new relation without changing any of its qualities? So far as I can see, the question has no precise answer, because the distinction between a quality and a relation is not precise. It is indeed easy to point out relations which no one would think of calling qualities, and *vice versa*; but the middle ground is not so clearly marked. Thus redness is a quality and nearness is a relation. But weight—is that a quality or a relation? Weight at the equator, to be more precise? I do not think that there is any definite answer. Clear thought is forever resolving qualities into relations with (of course) new qualities underlying them; as the mass of a body underlies its weight. With this proviso it seems safe enough to answer the question propounded—whether the relations of a thing may vary without change in any of its qualities—in the negative. At the same time it must be remembered that some relations are very superficial, and the qualities which they induce are very superficial also. Sometimes, however, we are urged to believe that the least change in any of the relations of a thing must involve some change in all its qualities. I see no reason why we should accept this. A change of place may or may not bring about a change of color: though, to

be sure, if the surface of the object were large and the movement were considerable, it would require an extraordinary concomitance of circumstances to make hue, tint, and saturation at every point exactly the same as before.

More important are the questions which arise in connection with the third form of the doctrine; but I shall have to discuss them with similar brevity. In recent controversy the question of the externality of relations has frequently taken on a new meaning: Are relations, or may they be, external to each other, *i. e.*, independently variable?

It is sometimes urged as a very weighty consideration, that spatial relations are largely independent of each other; for example, that a point may change its distance from a given point without changing its distance from a given line; and sometimes neo-Hegelians seem bound to contest statements of this sort. To these thinkers a change in one relation suffices to make the term in some degree a different term; and how can different terms stand in the same relation? This neo-Hegelian position appears to me to be utterly unsound. It is simple and innocent enough to hold that our conceptions of relations are not wholly clear and satisfactory, and consequently that no relations, as we conceive them, truly exist—simple and innocent, but futile; for a complete scepticism of relations is tantamount to a universal scepticism. But when relations, such as distance from a line or a point, are once assumed as truly existing; then to question their apparent mutual independence is more than futile. It is playing fast and loose with the facts. If points are points, and lines are lines, and points are distant from lines and from other points, then it is sheer caprice to question the proposition that two points may stand at the same distance from a given point and at different distances from a given line. Such questionings have no scientific or philosophical significance.

So far, then, the externality of relations is obvious enough. But are we entitled to go further? It is to be observed that if some relations are mutually independent others are quite as clearly interdependent; *e. g.*, the distance between two given masses and the attraction between them. Furthermore a few

given relations are sometimes sufficient to determine a whole class of relations; as the distance of a point from three given planes entirely determines its spatial relations. The question arises, whether the relations in which a thing stands may not be divisible into distinct groups or systems, between which an entire independence exists. It is to be observed that a somewhat analogous question arises in the case of qualities. How far are these external to each other? It seems clear that some qualities may vary independently of some others—the pitch and intensity of a tone, for example—while some are more or less definitely inter-connected. It has often been held that simple qualities are all equally compatible with one another. Bacon, for example, supposed that all the characteristics of any substance, such as gold, were reducible to a few simple forms; and that if one knew these forms and possessed a technique for bringing them about severally, one might change any substance into any other. One might give lead one by one the forms which characterize gold, and then the one-time lead would be gold. That the qualities of any concrete object are analyzable into such forms no one in our day would seriously suggest; though it must be confessed that the actual interdependence of qualities is (as Locke observed) only slightly known to us. I venture to suggest that much the same account must be given of the mutual implications of relations. That a change in any one relation or determinate group of relations, in which a concrete object stands, might take place without affecting *any* of its other relations, is an enormous assumption, which we have no motive whatsoever for making; though just what the detailed interconnections are, we must generally wait for experience to inform us.

To the bald question, whether relations are external or not, I do not see that any single answer can be given. All depends upon what is meant. That relations in general form no part of the essential nature of real beings, is, I think, clearly false. That some relations are unessential to some real beings is true if unessential has any acceptable meaning at all. That relations are external to qualities is, again, a vain presumption; but we have no reason to suppose that every relation is bound up with every

quality. Finally, the mutual independence of relations is a matter of more or less, which must for the most part be empirically determined.

I wish to add in this place a few words upon the prior question, as to the distinction between the essential and the non-essential. It has, I think, three different, but closely connected, meanings.

1. 'Essential' may mean, relevant to the accomplishment of a particular purpose or set of purposes. In this sense, what is essential in a horse depends upon what you wish to do with him—work him, race him, eat him, or sell him. What is essential for one purpose may be utterly non-essential for another. This is the sense upon which pragmatist writers have generally insisted. It has, however, in my opinion, a very limited importance in logical discussions.

2. 'Essential' may mean essential to a concept; that is to say, necessary to its discrimination from other concepts; or, if the concept can be defined, contained or implied in its definition. The peculiar efficiency of conceptual thought rests upon the degree in which it remains constant despite changes of purpose. The horse of science is a horse irrespective of your hopes and fears. Or, if we say with the pragmatists that a horse is always a horse only for the purpose of logical classification, we must remember that this purpose is not simply one among others—that it is not only an end in itself, but a means adapted indifferently to all other possible purposes.

It is notorious that many of our concepts are too vague to admit of exact definition. They are doubly vague; first, because the characteristics which they include are themselves confusedly understood; and secondly, because few if any of these characteristics are inseparably included. *The concepts are not of logical species, but of types*, which admit of an indefinite amount of divergence in all manner of directions. Such concepts, moreover, are peculiarly liable to change, and, in particular, to development. With reference, therefore, to such concepts, the term 'essential' does not admit of a perfectly precise application. It is a matter of more or less, and, even at that, admits of no precise measure.

The philosophical criticism of common sense has generally consisted in pointing out the vagueness of its concepts; and it has been the perennial hope of the system-makers that the whole body of the sciences, or at least some select portion, might be purged of all confusion. It is evident, however, that only a very limited realization of this hope has been attained. Vagueness, to be sure, is a matter of degree; and the concepts of science are, in general, far more definite and constant than those of common sense. But, with the possible exception of pure mathematics, all the sciences contain among their fundamental concepts some of which no precise account can be given; and this is probably not less true of epistemology and metaphysics than of any of the special sciences.

Where a concept can be defined, or where its meaning is determinable by means of a set of axioms, the distinction between essential and non-essential becomes fixed. In this connection it is worth while to remember that in the mathematical sciences definitions are wholly superfluous except as time-savers. All the propositions of mathematics can be directly expressed in terms of the indefinables. In fact, in this field definition is not primarily of concepts but of symbols. As Couturat has expressed it, a mathematical definition "is a logical equality (an identity), of which the first member is a new sign which has as yet no sense, and of which the second member, composed of *known* signs (among which, therefore, the sign to be defined does not appear), determines the sense of the sign in question. . . . A definition is *not a proposition*, for it is neither true nor false; it cannot be proved or disproved; it is a *convention* which has to do simply with the employment of a simple sign in place of an assemblage of signs." The distinction between essential and external is therefore in this field perfectly trivial except as applied to the indefinables; and as applied to them it amounts only to the distinction between theorems that are, and those that are not, demonstrable from the set of axioms that constitute the meaning of the indefinables. Thus it is essential to the Euclidean point that it be not a member of two such sets of points as are commonly called lines parallel to the same line; but it is external to it that

the set of points constituting a segment shall contain points corresponding to all real numbers. A new axiom, the Cantor-Dedekind axiom, may be added to cover this property, as is done in the theory of sets of points. But then the meaning of 'point' has been enlarged; and what was before external to it is now essential.

3. 'Essential' may mean essential to a reality (the sense called for in the first part of this paper); which is as much as to say, essential to a final concept of the reality, a concept which need nevermore be modified. But it is to be noted here, that it is only as an afterthought that we can reflect upon our possible or probable degree of ignorance, and thus formulate the distinction between what is essential to the concept as such and what is essential to the reality which the concept represents. In the actual employment of concepts, the two meanings coalesce. To think a concept and to think of a reality are the same.

Whether or not we possess, or can hope to possess, finally satisfactory concepts is a further question, which I cannot discuss here. But if we do possess them it is again within the limits of pure mathematics that they are alone to be found. And on this assumption what has been said above as to the concepts of mathematics applies directly to the realities which these concepts represent.

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