LECTURES ON PHYSICAL OPTICS

PART I

(SAYAJI RAO GAEKWAR FOUNDATION LECTURES)

BY

SIR C. V. RAMAN
Nobel Laureate in Physics

PUBLISHED BY
THE INDIAN ACADEMY OF SCIENCES, BANGALORE
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PREFATORY NOTE

In the month of February 1941, the author visited Baroda and delivered a course of two lectures on 'Light as Wave-motion' and 'Light as Corpuscles' respectively. It was the desire of the Foundation which invited the author to Baroda that the subject-matter of these lectures should be developed and written out in the form of a series of six lectures for publication. It was planned that the lectures would deal with the following topics: (I) Interference of Light, (II) Diffraction of Light, (III) Coronae, Haloes and Glories, (IV) Optics of Heterogeneous Media, (V) Light in Ultrasonic Fields and (VI) Molecular Scattering of Light. These topics had been the field of investigation by the author and his collaborators for many years and it was intended that the principal results of those investigations should find a place in the published lectures.

The preoccupations of the author slowed down the writing up of the volume for publication and finally brought it to a stop in the year 1943 after 160 pages had been printed off. Much labour and thought had been devoted to the work and it is believed that it contains material of enduring value and interest. Accordingly, it appeared desirable to release the part already printed as Part I of the lectures and thus make it available for perusal by those interested in optical theory and experiment.

LECTURE I

INTERFERENCE OF LIGHT

LIGHT is a phenomenon which we perceive and which plays a fundamental role in human life and activity. Constant experience makes us familiar with various aspects of the behaviour of light. These may broadly be classified under three headings. The first group of experiences relates to the geometric aspects of the propagation of light. Under this heading come the rectilinear path pursued by light from the source to the observer in free space, the casting of shadows by obstacles and the geometric laws of reflection and refraction at the boundary between different substances. The second group of experiences relates to the character of the sensations produced by light, which are threefold, namely, the brightness of light, the colour and the degree of its saturation. The third group of experiences connects light with the properties of material bodies, namely their capacity to emit, absorb, reflect, refract and scatter light, thereby making themselves visible. The study of the phenomena of light under these headings respectively constitute the three great divisions of optical science, namely, geometrical, physiological and physical optics.

The three categories of optical experience defined above can be brought into intimate relationship with each other only through an understanding of the ultimate nature of the emanation which we perceive as light. Experimental studies enable us to distinguish between those phenomena which are of a subjective or physiological nature and the properties of light that have a definite physical basis. The spectroscope, for example, enables us to separate out the rays of light of different colour and warns us that light which is perceived as yellow visually is not necessarily the same as the light which appears as yellow after spectral analysis. The spectroscope, in fact, enables us to make the first real step in understanding the physical nature of light. It indicates that there are different kinds of light which are physically different, yet analogous to each other, and that if optics is to be an exact science, we must consider the behaviour and properties of light which is truly monochromatic, in other words appears as a sharp single line in the spectrum. Fortunately, various sources of light are available in which the luminous centres are gaseous atoms, the radiations from which on examination through a spectroscope appear as discrete lines of sufficient intensity to be practically useful. Amongst these, the mercury vapour lamp is by far the most generally useful; it is indeed a veritable Alladin's lamp for the student of optics. Other sources of light are occasionally needed for special purposes. Amongst these may be mentioned specially the zinc amalgam lamp, which gives three lines in the blue region of the spectrum which are highly monochromatic, two of them being fairly intense. The lines of the mercury arc appear sharp and single when examined through an ordinary prismatic spectroscope. But examined through a high resolving-power instrument such as a Fabry-Perot etalon, a Lummer-Gehrcke plate or a reflection echelon grating, the mercury lines exhibit numerous components or satellites, while the lines due to the zinc atoms appear as truly single or monochromatic. In Fig. 1 (*a*) is illustrated the spectrum of the zinc amalgam lamp as recorded by an ordinary spectroscope. Fig. 1 (*b*), 1 (*c*) and 1 (*d*) record respectively the same spectrum as further analysed by a Fabry-Perot etalon, a Lummer-Gehrcke plate and a reflection echelon grating. Each of the lines is marked with its approximate wave-length in Angstrom units ($\mathring{A} = 10^{-8}$ centimetre), the symbols Hg and Zn denoting that the radiations are due to the mercury and zinc atoms respectively.

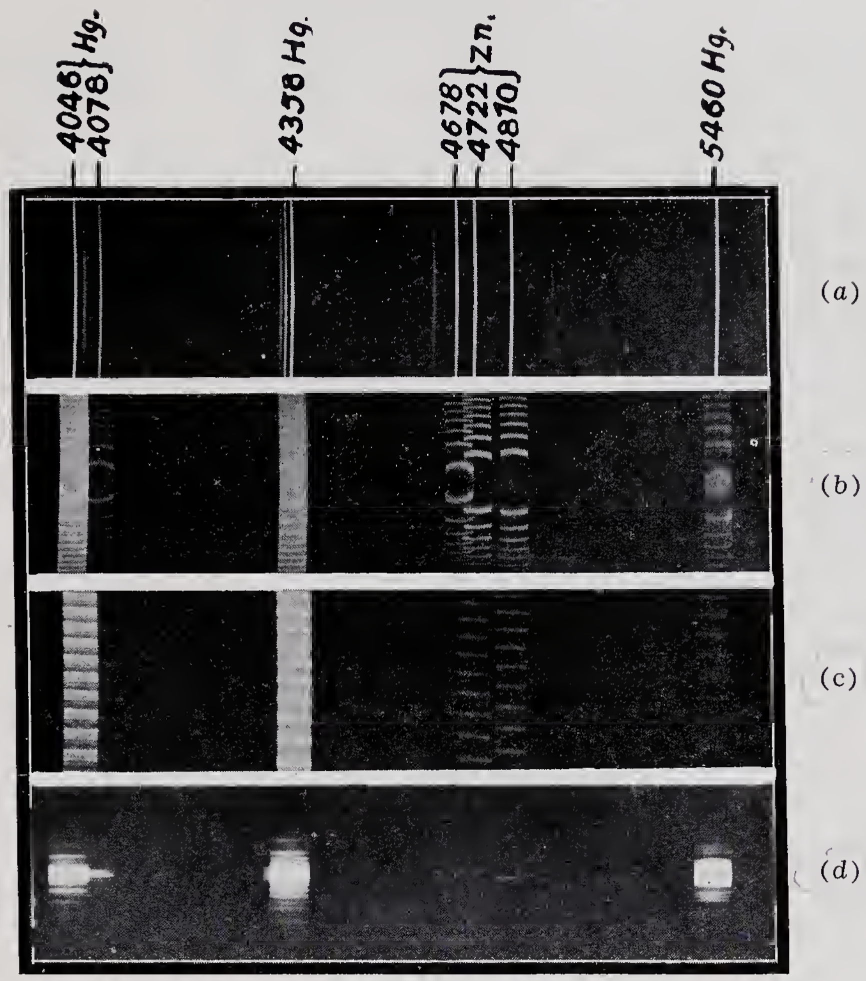

Fig. 1

Spectrum of Zinc Amalgam Lamp:
(a) Prism Spectrograph
(b) Fabry-Perot Etalon
(c) Lummer-Gehrcke Plate
(d) Reflection Echelon Grating

The Interference of Light.—The geometric theory of light rays forms the basis of applied optics and is extensively and successfully employed in the computation and design of the lens systems used in optical instruments of various kinds. The practical use made of these instruments in the field and in the laboratory also assumes the validity of the laws of geometrical optics, including especially the rectilinear propagation of light in uniform media. The wave-theory starts with a different view of the nature of light, namely that it is wave-motion propagated through space and that monochromatic light has associated with it a definite frequency of oscillation and a definite wave-length, the product of the two being equal to the wave-velocity in the medium. In certain simple cases, namely in the cases of plane and spherical waves in an isotropic medium, the relation between the ray and wave concepts of light is readily stated; the direction of the geometric rays in these cases is identical with the direction of movement of the wave-fronts. The essentially new possibility which the wave-concept involves is that of interference, *viz.*, that when two beams of light originally derived from the same light source are superposed on each other, the light intensity at any point in the field may be either greater or less than the arithmetic sum of the intensities due to either separately. That such an effect is actually observed forms the strongest support for the wave-theory of light.

We may illustrate the principle of interference by considering the case of two beams of light which are divided from the same original beam by suitable optical arrangements and which traverse together a limited region of space before again separating. It will be assumed that the light beams

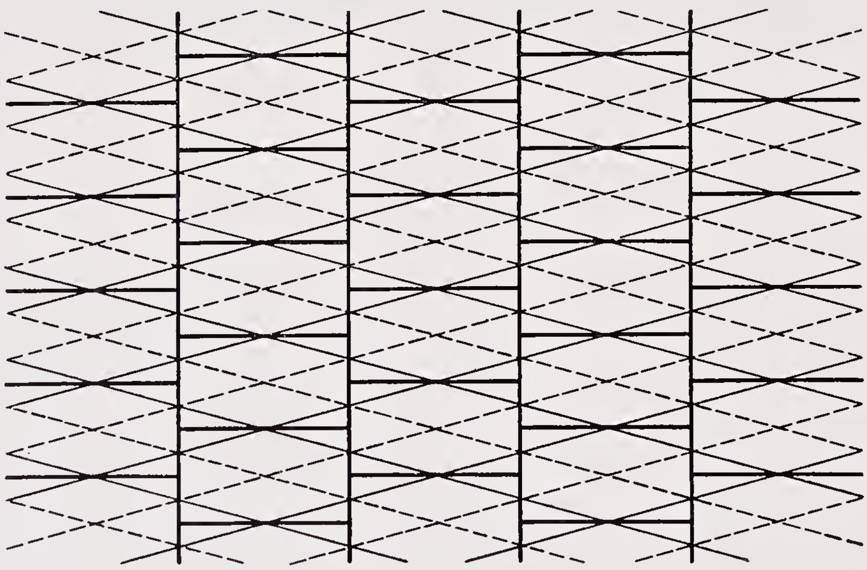

Fig. 2
Interference of Plane Wave-Trains

consist of polarised light of wave-length λ and that the directions of their travel lie in the plane of the paper and cross each other at an angle 2Ψ (Fig. 2). The amplitudes and directions of the light vector in the two wave-trains may be assumed to be identical and to be normal to the plane of the paper. The thin oblique lines in the figure represent the wave-fronts in which the light vector at a particular epoch is a maximum upwards, while the broken lines represent the intermediate planes at which it is a maximum downwards. The thick lines which have been drawn bisecting the obtuse angles between the wave-fronts represent planes in which the light-vectors are in opposite phases and the resultant intensity is, therefore, zero. At the intermediate planes, the intensity would be a maximum. The thick lines bisecting the acute angles between the wave-fronts are the planes along which the resultant light vector at the given epoch is a maximum upwards. The spacings of these two sets of planes are given by the relations $\lambda = 2D \sin \Psi$ and $\lambda = 2d \cos \Psi$ respectively. When Ψ is zero, D is infinite and d is $\lambda/2$, while when $\Psi = \pi/2$, D is $\lambda/2$, and d is infinite. It is evident that the amplitude of the resultant disturbance oscillates as we pass along the horizontal lines in the figure, while its phase is reversed whenever we cross any of the vertical lines of zero intensity. Hence, the horizontal lines cannot be considered as representing true wave-fronts and they do not therefore possess any significance from the standpoint of geometrical optics. This will be further evident when we recollect that when the two wave-trains separate, they pursue their courses independently along their original directions of propagation. The alteration of the energy distribution in the field indicated by the principle of interference is thus not in any way a contradiction of the basic ideas of geometrical optics.

It is evident from Fig. 2 that the effect of interference between the two sets of plane waves is to produce a stratification of intensity in the medium, the spacing of which is very wide when the inclination between the wave-fronts is small and diminishes as the inclination increases, reaching the limiting value $\lambda/2$ when the waves travel in opposite directions. When the stratifications of intensity are widely separated, they are readily seen as interference bands in the field. The observation becomes less easy and needs special technique when the spacing of the stratifications narrows down and approaches the limiting value of half the wave-length of light. The optical conditions represented in the figure can be experimentally realised in a variety of ways. The simplest method is to obtain one of the two beams by reflection at the surface of a mirror at the desired angle of incidence, while the other beam is furnished by the incident light itself. It is readily seen from the figures that the character of the resultant disturbance would, in general, be very different when the incident light is polarised with the vibrations respectively parallel and perpendicular to the plane of incidence of the light on the mirror. In the latter case, which is the one discussed above, the light vectors in the interfering wave-trains would be parallel to each other and would differ only in phase, and the alternations in intensity resulting from interference would therefore be most noticeable. On the other hand, when the light vectors lie in the plane of incidence, they would be parallel to the wave-fronts and would therefore be inclined to each other in the two wave-trains. The resulting disturbance would therefore, in general, be elliptically polarised. It is evident that except in the cases of nearly normal or nearly grazing incidence of the light on the mirror, the interferences in this case would result in less conspicuous variations of intensity than when the light vectors are perpendicular to the plane of incidence.

Interferences of Parallel Plates.—As remarked above, the spacing of the stratifications of light intensity in an interference field depends on the angle at which the wave-fronts cross; the larger the angle, the less easily noticeable would they be. The optical conditions for observing the results of interference would obviously thus be most favourable when the superposed beams of light are completely coincident in direction, as the intensity of illumination over the entire field would then be enhanced or diminished, and it would be possible also to use an extended source of light. Such a situation arises when a pencil of light is reflected by or transmitted through a plate of transparent material bounded by plane parallel surfaces. In the light reflected by such a plate, a series of successive reflections at its surfaces appear superposed, the first external and internal reflections being the two strongest and nearly equal in intensity. Similarly, the light beam transmitted by the plate has superposed on it the light beams which have suffered an even number of internal reflections within the plate, these being much weaker. It is readily shown that the optical difference of path between the successive superposed beams in either case is $2\mu t \sin \theta$ where μ is the refractive index and t the thickness of the plate, and θ is the glancing angle of internal reflection. The reversal of phase which occurs at an external reflection has also to be taken into account. Accordingly, in the light reflected by the plate, we have the minimum intensity if $2\mu t \sin \theta = n\lambda$ where n is an integer and λ the wave-length of the light in vacuum. The same condition gives the maximum intensity for the transmitted light.

The considerations stated above indicate that the intensities of the light reflected and transmitted by a plate should exhibit fluctuations if either the thickness of the plate or the angle at which it is viewed be varied. Interferences of this kind are very readily observed and were indeed historically the first to be noticed and explained on the principles of the wave-theory. If the plate be sufficiently thin, white light may be used for the observations, the alternations of intensity then manifesting themselves as variations in the colour of the reflected or transmitted light. The colours of soap films, for instance, arise in this way. If a flat soap-film be set vertically, the horizontal bands of colour which develop on its surface are the result of the thickness of the film increasing as we go downwards. Colour bands on a soap-film may, however, also arise from a variation of the angle at which its surface is viewed by the eye. This effect is well shown by a spherical soap-bubble; if the bubble is blown of a uniform thickness, the colour bands on its surface are due solely to the varying obliquity of observation and appear as concentric circles around the line of sight, irrespective of the direction from which the sphere is viewed. On the other hand, if a bubble be of non-uniform thickness, the distribution of colour depends on the direction from which the bubble is viewed. The colour bands are horizontal circles when the bubble is viewed from above or below. But when the bubble is viewed horizontally, the circles appear deformed or displaced downwards, as the result of the effects of varying thickness and of varying obliquity of observation appearing in combination. A spherical bubble at rest tends to drain downwards. This may however be counteracted and the bubble maintained in a state of uniform thickness by gentle currents of air impinging on its surface.*

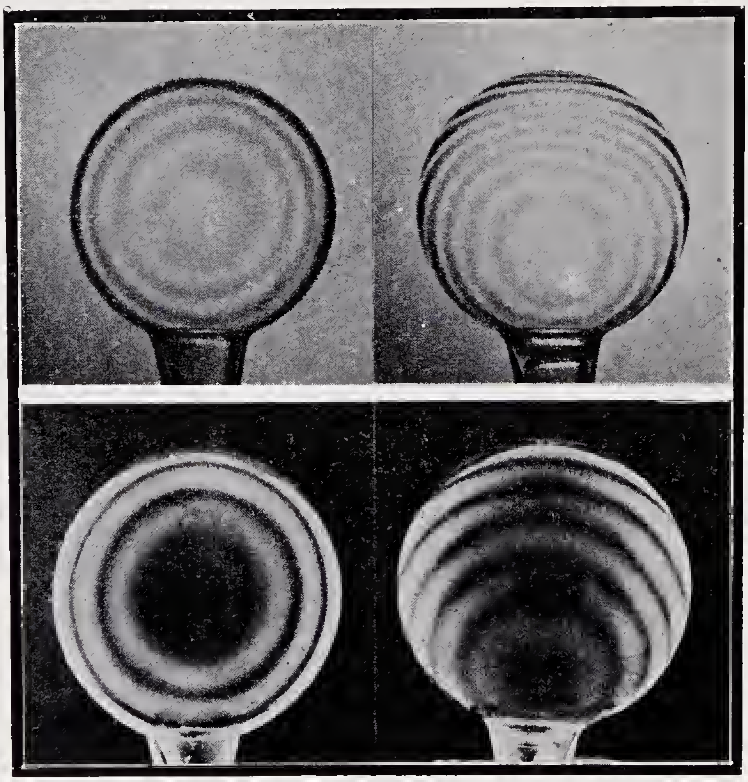

FIG. 3
Soap Bubbles in Monochromatic Light

The phenomena referred to above are illustrated in Fig. 3, the two upper pictures being respectively those of

* C. V. Raman and V. S. Rajagopalan, *Proc. Ind. Acad. Sci.*, 1939, 10, 317.

a uniform and non-uniform bubble viewed horizontally by transmitted light, while the two lower pictures represent similar bubbles by reflected light. The figures also illustrate other features of interest. It will be noticed that the interferences as seen in transmission and by reflection are complementary in appearance. This is to be expected, as the energy that disappears in reflection appears as transmitted light, and *vice versa.* The dark and bright rings in the transmitted light, therefore, correspond respectively to the bright and dark rings seen by reflection. The contrast between the dark and bright rings by transmitted light is evidently much less than in reflection. This is also to be expected, as the interfering beams are not of comparable intensity in the former case, whereas they are of practically equal intensity in the latter. It will be noticed, also, that the contrast between the dark and bright rings by transmitted light rapidly increases towards the margin of the bubble. This is due to the increased reflecting power at oblique incidences which makes the intensity of the interfering beams much more nearly equal. Very near the margin of the sphere, the dark rings as seen by reflected light are much sharper than the bright rings, while by transmitted light we have the opposite effect. This is due to the influence of multiple reflections within the film which tend to sharpen the interference bands, a principle which is utilised in the Fabry-Perot etalon and the Lummer-Gehrcke plate.

Haidinger's Rings.—The interference colours exhibited by parallel plates in white light naturally cease to be visible when the plate is not very thin. The interferences may, however, be observed with thick plates if we use monochromatic light. The case in which the plate is of uniform thickness is of particular interest, as the fluctuations of intensity would then be solely due to variations of the angle of incidence at which the plate is viewed. If an extended source of monochromatic light is seen by reflection at the surfaces of such a plate, the eye being adjusted for distant vision, a set of dark circles would be seen at infinity in the directions corresponding to the values of θ for which the formula $2\mu t \sin\theta = n\lambda$ is satisfied, while bright circles would be seen in intermediate directions. It is evident from the formula that the rings would be centred around the direction of the normal to the plate. As also shown by the formula, they would appear widely separated in the vicinity of the normal, would crowd up in more oblique directions, and open out again in directions nearly parallel to the surface of the plate which correspond to nearly critical incidence of the light within the plate.

The theory of these rings was implicit in the explanation of the colours of thin plates given by Thomas Young in 1809. They were, however, first observed by Haidinger in 1849. It is obvious that to enable them to be seen perfectly in all circumstances, the thickness of the plate should be rigorously constant. In the earliest observation of the rings by Haidinger, this condition was realised by the use of a natural cleavage sheet of mica, the yellow flame of a lamp with salted wick being viewed by reflection at its faces.* The Haidinger rings are of great interest in optics, as they are utilised for the spectroscopic examination of light in the Fabry-Perot etalon and the Lummer-Gehrcke plate, and also furnish the theoretical background for the interferences observed in other instruments, *e.g.*, the Michelson and the Jamin interferometers. In these applications, it is necessary that the plates used should be thick and uniform, and their preparation, therefore, requires a high degree of technical skill. So much emphasis is usually laid on this point that the impression naturally prevails that optically worked plane-parallel plates of glass are essential for the observation of the rings. This impression is, however, not justified. Actually, the rings can be seen in any ordinary plate of glass and it is not necessary that it should be uniform or even plane.‡ The possibility of observing the rings under these conditions

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* See also T. K. Chinmayanandam, *Proc. Roy. Soc.*, A, 1918, **95**, 176.

‡ C. V. Raman and V. S. Rajagopalan, *Phil. Mag.*, 1940, **29**, 508.

depends upon limiting the area of the plate used to such extent as may be necessary. This may be accomplished using a diffusing screen of smooth board, white in front and blackened behind, with a small circular aperture at its centre. The screen is held very close to and behind the plate of glass with which the rings are to be observed, the white side facing the plate and illuminated by the light of a mercury lamp. The rings are then seen against a dark background by the observer's eye placed behind the aperture in the screen (Fig. 4). If the plate is both thick and non-uniform, the circular aperture in the screen may be replaced

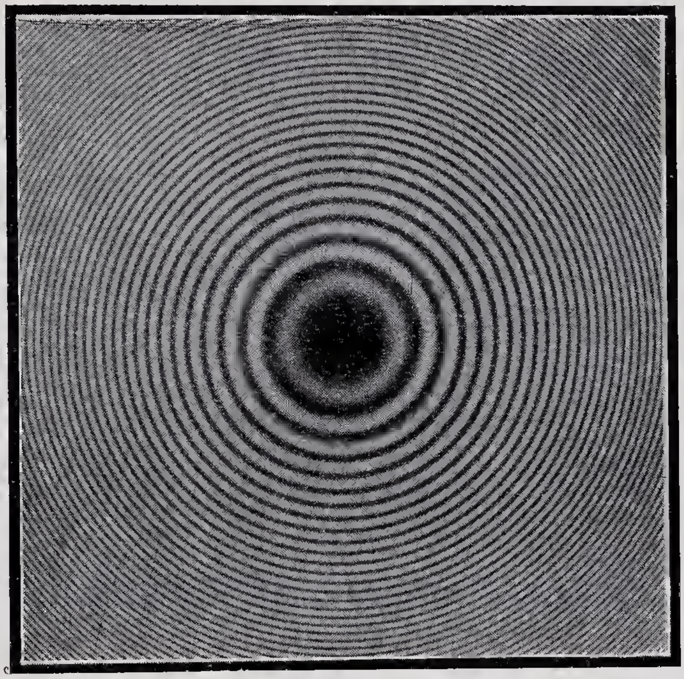

FIG. 4
Haidinger's Rings in a Glass Plate

by a fine slit which can be turned round and set parallel to the contour lines of uniform thickness of the plate. A section of the ring-system is then seen in perfect definition along the length of the slit. Fig. 5 shows such a system of interferences photographed with a plate of glass 3·5 millimetres thick and so non-uniform that distant objects exhibited by reflection in it two distinct images of varying separation.

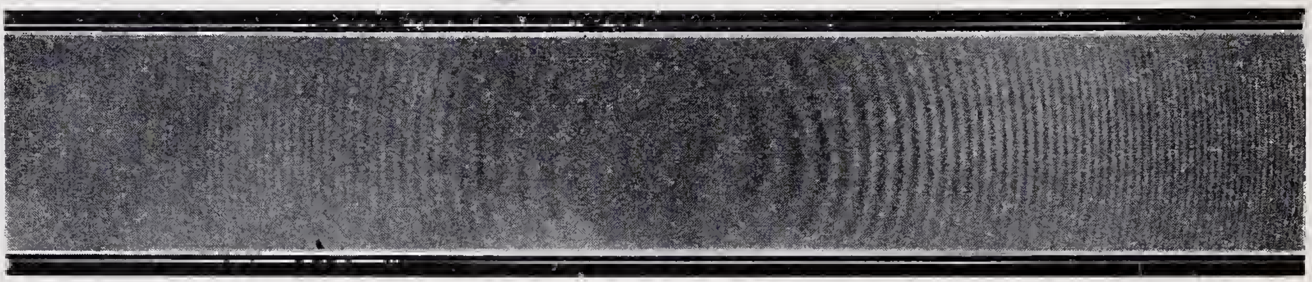

FIG. 5
Haidinger's Rings in a Non-uniform Plate

If with the arrangements described above, the plate is moved away from the viewing screen, the rings gradually transform to the interferences of the Newtonian type due to the varying thickness of the glass plate. These are located at or near the surface of the plate, while the Haidinger pattern for a flat plate is located at infinity. It is also

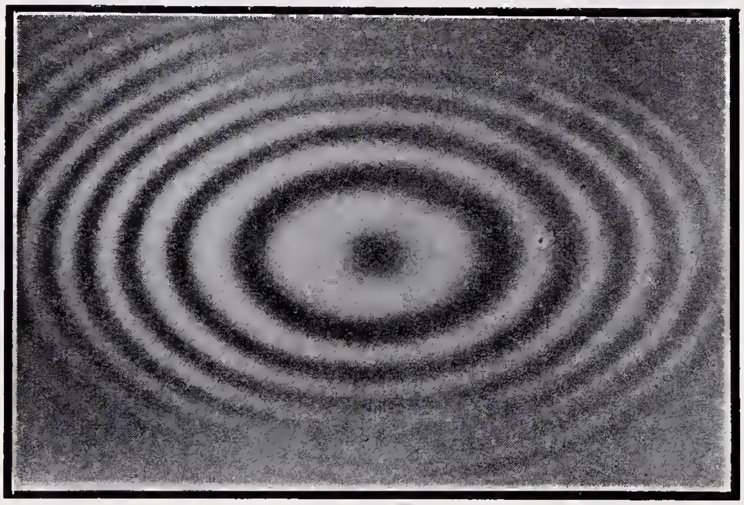

FIG. 6
Haidinger's Rings in a Cylindrical Plate

possible to observe the Haidinger's rings in a curved plate of mica* with the viewing arrangement described above. The configuration of the rings then depends on the distance of the eye from the plate. If the plate be uniform in thickness, the pattern seen is determined solely by the variation of the obliquity with which the surface of the plate is viewed by

* C. V. Raman and V. S. Rajagopalan, *Jour. Opt. Soc. Am.*, 1939, 29, 413.

the observer, and it is readily verified that it is equivalent to the normal Haidinger system modified in the same way as the image of a set of concentric circles would be, if seen by reflection at the curved surface of the plate. (See Fig. 6.)

The Fabry-Perot Etalon.—The Haidinger rings in a plate half-silvered on both sides and seen by transmitted light form the essential principle of the Fabry-Perot etalon which, as already mentioned, is a most useful and powerful appliance for the analysis of light. A very great improvement in the appearance of the fringes seen in transmission is effected by the half-silvering which makes the intensities of the interfering beams much more comparable, and as the result of multiple reflections, also largely increases their number. In practice, it is found more convenient to use a plate of air enclosed between two optically worked plane surfaces of glass; these surfaces are half-silvered and kept strictly parallel at a suitable distance apart by a separating ring of invar metal. The Fabry-Perot rings, as they are called, are observed when an extended source of monochromatic light is viewed in transmission through the plate. If the light used be highly monochromatic, they are seen as sharp bright circles on a dark background. As the result of the silvering and of the multiple reflections resulting therefrom, the transmitted light is much enfeebled and its intensity is negligible except in the precise directions for which all the emerging beams differ in path by the same integral number of wave-lengths and can, therefore, totally reinforce each other. For a thick plate, these directions vary very rapidly with the wave-length, and the rings corresponding to closely spaced spectral components in the radiation are, therefore, clearly separated.

It is naturally desirable to use etalon plates of a fairly large area, as more illumination is thereby secured, which is a matter of great importance when working with very faint sources of light. The utility of the etalon is greatly enhanced when the separation of the plates can be varied to suit the problem under investigation. This is conveniently effected by having a selection of metal rings of different thicknesses, the aggregate of which, together with the plates themselves, makes up the total space within which the etalon is mounted. An invar ring of the thickness desired is placed between the plates, while other metal rings fill the gap outside. The etalon plates are adjusted to perfect parallelism by a delicate mechanism which exerts the minimum pressure necessary to tilt and hold them in position. It is convenient to place the etalon between the source of light and the slit of the spectrograph. An image of the interference pattern is focussed carefully on the slit, and the adjustment of the etalon is made by trial till the best definition of the rings is obtained. The slit of the spectrograph is kept wide open so that a section of the ring pattern is recorded on the photographic plate, the different spectral lines, however, being separated by the dispersion of the spectrograph. Different etalon separations may be used to check the order of interference corresponding to any particular ring seen in the pattern. The absolute wavelength of the radiations can also be determined exactly from the positions of the rings, if the etalon separations are known.

The Jamin Interferometer.—This very useful instrument which has been extensively employed in refractometry is an application of the phenomenon known as Brewster's Bands. Sir David Brewster observed coloured interference bands crossing the image of a source of white light seen by reflection successively at the surface of two plates of glass of equal thickness. The width of the fringes decreases with increasing inclination of the plates to each other.

The course of the interfering beams in the Jamin instrument is shown in Fig. 7. Part of the light incident on the first plate is reflected at its front surface, and then at the rear surface of the second plate; another part is reflected at the rear surface of the first plate and then at the front surface of the second. The paths of these two beams are equal, irrespective of the angle of incidence, provided the plates are of the same thickness and parallel to each other. If, however, the plates are inclined to each other, the paths are equal only when the incident beams make equal angles with the two plates; for other directions, the path difference would progressively increase. The appearance of fringes with an achromatic centre, and with a width diminishing with increasing inclination of the plates is, therefore, readily understood.

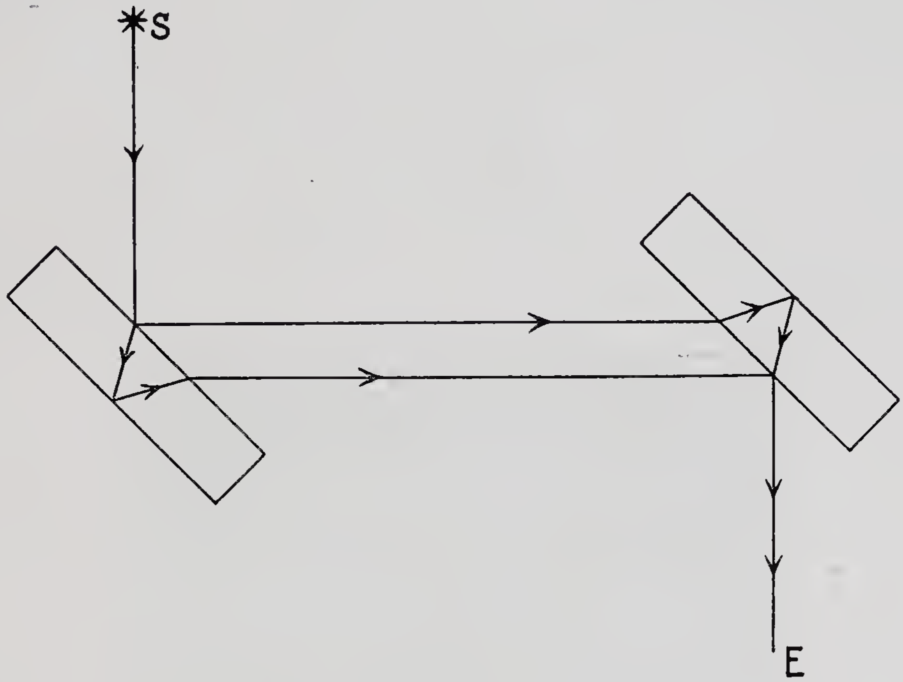

FIG. 7
Principle of Brewster's Bands

Brewster's bands can, of course, be seen also with monochromatic light, and indeed the observations can be pushed much further with it. Ketteler, Lummer and others have studied the form of the interference figures over a wide range of incidences and of inclinations of the two plates relative to each other and also for plates of unequal thickness. The figures observed fall into two classes. These correspond to the cases in which the interference occurs under a path difference equal to the sum and the difference respectively of $2\mu_1 t_1 \sin\theta_1$ and $2\mu_2 t_2 \sin\theta_2$, these quantities being the relative retardation of the light beams reflected at the front and rear surfaces of each of the plates separately. It will be noticed that these quantities determine the Haidinger patterns of the two plates, and this suggests an alternative and very instructive way of regarding the theory of Brewster's bands.

It is evident from the diagram (Fig. 7) that the pattern seen by the eye placed at E and viewing an extended source at S is really the Haidinger system of reflected rings formed by the first plate and then again by the second, in other words, a multiplication of the intensities of the two systems.* The angular position of the dark rings in the two systems are given by the usual expressions

$$2\mu_1 t_1 \sin \theta_1 = n_1 \lambda \text{ and } 2\mu_2 t_2 \sin \theta_2 = n_2 \lambda.$$

The effect of the multiplications of intensities is to give a series of superposition figures,‡ which may be classified as differentials and summationals of the first, second and higher

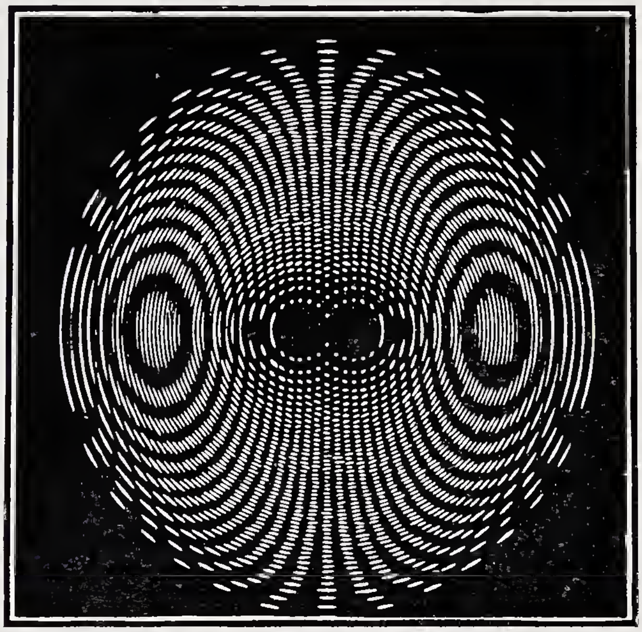

FIG. 8
Superposition of Haidinger Patterns

orders, and the form of which may be derived graphically or analytically. As the Haidinger rings are widely separated at normal incidence, and after first closing up, open out again at more oblique incidence, the complete first order differential pattern consists of two sets of closed curves, the configurations

* A. Schuster, *Phil. Mag.*, 1924, 48, 609.

‡ C. V. Raman and S. K. Datta, *Trans. Opt. Soc.*, 1925, 27, 52 and 1927, 28, 214.

of which relative to the Haidinger's rings are indicated in Fig. 8 for a particular inclination of two plates of equal thickness. It will be noticed that in the centre of the field, which corresponds to the symmetric direction, the fringes are straight. These are the Brewster bands observed in white light.

Optical Study of Percussion Figures.—We shall now consider as an example of interferences of the Newtonian type, a case arising in the study of the permanent deformation of plane surfaces by impact or static pressure. Hertz's well-known theory of impact was found to be a correct description of the facts in the case of spheres impinging on each other, only if their surfaces are smooth* and highly polished and the velocity of impact is sufficiently small. Accordingly, for an experimental test of this theory as extended to the impact of spheres upon flat plates,|| it was decided to study the collision of polished hard steel balls on smooth glass plates. It was then discovered that if the size of the balls or their velocity exceeded certain limits, the impact resulted in the production of percussion figures of beautiful geometric form in the glass plates.‡ A circular crack starts from the surface of the plate and spreads obliquely inwards in the form of a surface of revolution, revealing itself by the light which it reflects. The deformation of the external surface of the glass plate resulting from the collision is very conveniently exhibited by laying another flat glass plate on it, thus forming a wedge-shaped film of air between the two surfaces. The light of a mercury lamp passed by a green ray filter and incident nearly normally upon this film and reflected by it results in interferences which can be readily photographed, the camera being focussed on the percussion figure itself.§

* C. V. Raman, *Phys. Rev.*, 1918, 12, 442.
|| *Ibid.*, 1920, 15, 277.
‡ C. V. Raman, *Nature*, 1919, 104, 113.
§ C. V. Raman, *Jour. Opt. Soc. Am.*, 1926, 12, 387.

It is evident that the method for the study of percussion figures illustrated in Fig. 9 can be extended to all solids which are capable of being polished to have a smooth reflecting surface. An inspection of the photograph reproduced shows three distinct regions in the figure. Firstly, there is

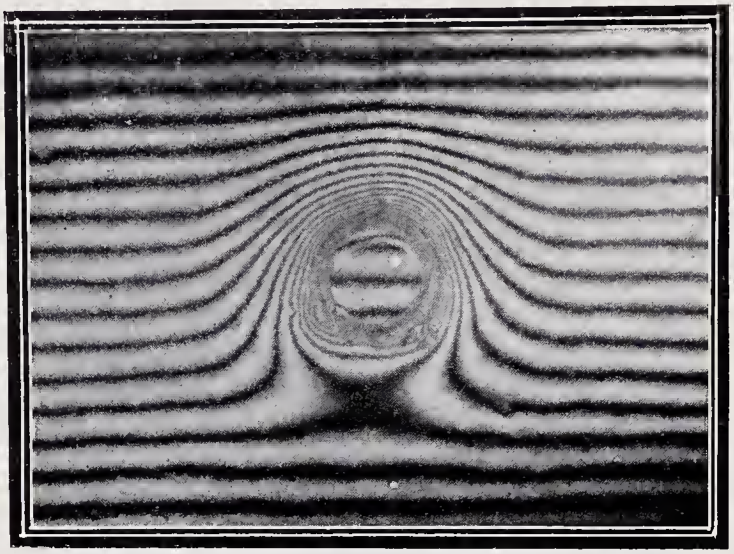

FIG. 9
Percussion Figure in a Glass Plate

a central area which is circular and is apparently unaffected by the impact, as is shown by the fringes passing through it being straight and parallel. Secondly, there is a narrow annular region of fracture full of a network of irregular fringes, showing severe injury to the surface. Thirdly and just beyond this, there is a sudden elevation of the surface which slopes down, first quickly and then more slowly, to the original level of the surface at the edge of an area which sets the limit to the percussion figure. Closer examination reveals another remarkable feature, namely, that the central area of the percussion figure, though it remains plane and apparently undisturbed, has, in reality, been *depressed below* the original level of the surface by an appreciable fraction of a wavelength, as shown by the fact that the course of the fringes outside the percussion area and within the central circle are distinctly out of register. This feature is observed in the percussion figures even with very thick plates of glass.

Additional details are given in a paper by K. Banerji,* where further work is reported on with glass and metal plates. A brief note has also appeared on the percussion figures of crystalline rock-salt photographed by the same method.‡

Visibility of Interferences in White Light.—The colours exhibited by thin films illustrate the general principle that when the path-differences are sufficiently small, the effects of interference are perceptible to the eye even with white light. In all interference experiments, if the path-difference is zero and the maxima or minima of illumination for all wave-lengths are therefore coincident at some point in the field, an achromatic fringe is there seen, appearing white or black as the case may be. Further out, the fringes are appreciated by the eye principally as alternations of colour, some six or seven of these being seen of gradually diminishing clearness. The explanation of these effects becomes clear when the interferences are examined with a spectroscope with the slit set parallel to the fringes, so that the interferences appear as dark and bright bands crossing the spectrum. The number of such bands is small when the slit is near the achromatic fringe, but increases rapidly as the slit is moved away from it, till ultimately the bands are distributed over the spectrum with some approach to uniformity. The failure of the eye to appreciate any differences of colour or intensity in such circumstances is not surprising.

The number of orders of interference visible in white light may be greatly increased by causing the interfering beams to traverse approximately equal optical paths in two media of different dispersive powers. We may, for instance, using white light, interpose a glass plate in one of the arms of a Michelson interferometer and adjust the air path in the other arm to approximate equality. Several hundreds or even some thousands of interferences may then be seen and enumerated, the number depending on the thickness of the glass plate and its dispersive power, but the fringes are

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* K. Banerji, *Ind. Jour. Phys.*, 1926, 1, 59.

‡ S. Smith, *Nature*, 1931, 127, 856.

visible only as very small alternations of colour in the field.* The explanation of this effect is as follows:—The addition of an optical path D in glass of refractive index μ (relative to air) in one arm of the interferometer and of an air path t in the other arm, changes the order of the interference at any point in the field by the number $(\mu D - t)/\lambda$, λ being the wave-length in air of a particular region in the spectrum. The position of the interferences in the field is therefore stationary for small changes of λ, if the variation of this number is zero, that is if

$$D(\mu - \lambda d\mu/d\lambda) = t. \qquad (1)$$

Equation (1) is equivalent to the statement that the retardation of a *wave group* produced by the extra path D in glass is exactly balanced by the additional air-path t. The corresponding change in the order of interference, in other words, the shift of the "achromatic" band measured by the number of fringes is

$$D . d\mu/d\lambda. \qquad (2)$$

If the glass plate be thick or if its dispersive power be great, this shift may be quite large. But since $d\mu/d\lambda$ alters with the wave-length, neither the thickness t of the compensating air-path, nor the order of interference $D.d\mu/d\lambda$ for which equation (1) is satisfied, is even approximately independent of λ. The interferences are therefore "achromatic" only for a limited region of the spectrum, and the position of the "achromatic" band in the field shifts with the part of the spectrum under consideration. In other words, the achromatic band is "dispersed" by the introduction of the glass plate and the number of interferences visible in white light is thereby increased enormously, but only at the expense of diminishing the visibility of the individual fringes almost to the limit. The extraordinary sensitivity of the eye to small differences in colour in adjacent areas, however, enables such fringes to be observed and enumerated.

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* N. K. Sethi, *Phys. Rev.*, 1924, 23, 69.

Sethi* to whom the foregoing explanation of the facts observed by himself is due, has shown that it covers also the increase in the number of interferences visible with white light in a plate of non-uniform thickness produced by viewing them through a dispersing prism—an observation originally made by Newton. Its correctness is proved by spectroscopic examination of the interference fringes at various parts of the field. It is then noticed that the interference bands are widely separated in a particular part of the spectrum and crowd together on either side of it. The region of the spectrum at which this effect is observed is found to vary with the part of the interference field under observation. Why the interferences are perceptible to the eye in spite of the great number of bands crossing the spectrum is thereby made intelligible.

Interferences in Polarised Light.—The light reflected at both the surfaces of a transparent plate bounded by the same medium is completely polarised at a particular angle of incidence on the external surface, and if viewed through a polariser set so as to transmit only vibrations in the plane of incidence, is completely quenched. It follows that the plate would, at this incidence, exhibit no interferences by transmitted light for the vibration parallel to the plane of incidence. When the light falls more obliquely, the interferences reappear in the parallel component, but would continue to be weaker than those in the perpendicular component until grazing incidence is reached. Holding up a plate of glass obliquely against a mercury lamp and viewing the interferences through a polariser, it is readily verified that in the fringes seen by *reflection, the maxima* are broader and more intense, while the minima are narrower and more sharply defined for the perpendicular than for the parallel component of vibration. In the fringes seen by *transmission* we have the complementary effect, the minima being broader and darker and the maxima narrower and more sharply defined in the same circumstances.

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* N. K. Sethi, *Proc. Ind. Assoc. Cult. Sci.,* 1921, 7, 37.

The cases where the plate is bounded by two different media present some special features of interest. Such a situation arises when a film has one free surface and a solid or liquid backing. The phenomena are then influenced by the optical properties of both the film and the backing material, as also by the nature of the transition between them. The reflections at the front and back surfaces may be of very unequal strength, and this would naturally affect the liveliness of the interferences. The incidence (if any) at which the light is polarised by reflection and beyond which a phase-reversal occurs for the parallel component of vibration would also, in general, not be the same for the two surfaces. Hence the circumstances in which interference occurs would, in general, differ widely for the two components of vibration. If such a plate be viewed through a polariser held in front of the eye and the latter is rotated, striking changes in the colour and intensity of the reflected light may be noticed depending on the obliquity of incidence. As a noteworthy example of this kind, we may mention the oxidation colours exhibited by a polished plate of copper which has been heated up in contact with air. The colours as seen in the parallel component are, in general, more vivid than in the perpendicular component; the two are also observed to be of a complementary character at sufficiently oblique incidences.

Soap-Bubbles between Crossed Nicols.—Some beautiful and interesting phenomena are noticed* when a soap-bubble is placed between two crossed nicols or polaroids and viewed by transmission against a bright source of light. When only one of the nicols is present, the usual colours by transmitted light are noticed, except that they are slightly more vivid towards the end of one diameter of the sphere and slightly less vivid at the ends of the perpendicular diameter. With both nicols present, a black cross appears on the surface of the bubble with its arms parallel to the vibration planes of the two nicols. Elsewhere, the surface of the bubble exhibits

* Unpublished observations by the author.

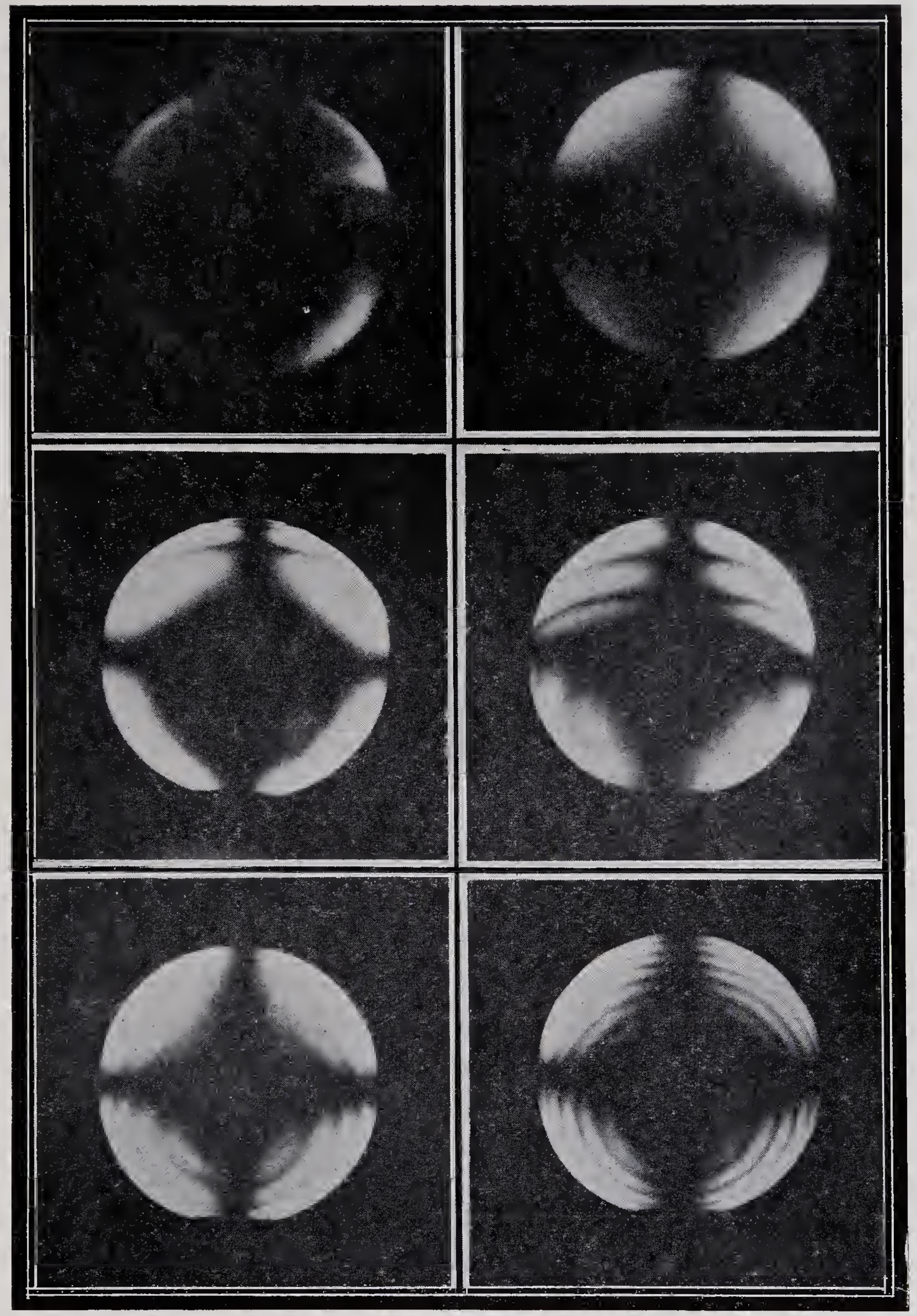

FIG. 10

Soap Bubbles between Crossed Nicols

striking colours which recall those seen by reflected light in their vividness and are, indeed, complementary to the usual transmission tints. With a monochromatic light source, the interferences are as striking as those ordinarily seen by reflected light. Near the margins of the bubble, the minima are sharp and much finer than the maxima, thus reversing the effects usually observed in transmitted light. As the bubble thins down, the interferences successively disappear so that in the penultimate stage its entire surface is bright except for the dark cross. When, finally, the soap-bubble goes "black", it retains a faint luminosity, while its spherical margin shines brightly as a crescent of light interrupted by the intersections with the black cross. These effects are illustrated in the series of six photographs reproduced in Fig. 10. These are arranged in order of increasing thickness of the soap film, the first being that of a bubble which has gone black near the top.*

When the nicols are set with their vibration planes not exactly at right angles, the black cross breaks up into two curved arcs or isogyres. These shorten and approach the margins of the sphere rapidly as one of the nicols is further turned round. With a thick film, the isogyres are themselves the most vividly coloured parts of the bubble. With thinner films it is noticed that when the nicol is turned so that the isogyre moves across an area of the bubble, the colour of the same alters to the complementary tint. With monochromatic light, the isogyres show notable alternations of intensity and appear distorted where they cut the interference curves, while the latter exhibit dislocations at these points which may amount to as much as half a fringe. A "black spot" on the bubble usually appears as a dark area on a bright background. But when it passes over one of the isogyres, its optical character reverses and it is then seen as a *bright spot on a dark background.*

* The so-called "liquid soap" which is commercially available can very conveniently be used for such experiments. Stable and uniform bubbles are obtained with a highly diluted solution of the same. This should be freshly prepared.

The explanation of these effects has been discussed in a paper by K. S. Krishnan* on the assumption that the films are optically isotropic.† When plane-polarised light is incident on the bubble, the coefficients of transmission and internal reflection are different for the components of the vibration parallel and perpendicular to the plane of incidence. Hence, except when one or other of these components vanishes, the plane of polarisation is rotated by transmission or internal reflection at the surfaces; the light vector turns away from the plane of incidence in transmission and turns towards that plane further at each successive reflection. These rotations are, of course, superposed when a beam undergoes transmissions and reflections successively. In general, therefore, the second polariser fails to extinguish the transmitted and reflected beams. Their components emerging from it accordingly interfere and give the observed luminosity. The smallness of the rotation of the light vector in the transmitted beam taken together with its much greater magnitude and its opposite sense for the light vector in the reflected beam makes the relative amplitudes of these beams after passage through the second polariser comparable, and also involves a reversal of their relative phases. The vividness of the interferences and their similarity to those ordinarily exhibited by the bubble in reflected light are thus readily understood. At nearly grazing incidences, the transmission coefficients become small and the reflection coefficients increase considerably, and the resultant rotations of the light vectors for the transmitted and multiple reflected beams are also no longer in opposite directions. Hence, the considerations stated above require some modification at nearly grazing incidences. Taking these circumstances into account, Krishnan has given an explanation of the luminous crescent exhibited by the "black" bubbles.

Haidinger's Rings in Crystals.—The natural cleavages of various crystals, *e.g.*, mica or gypsum, enable us readily to obtain transparent plates with good optical surfaces which are

* K. S. Krishnan, *Ind. Jour. Phys.*, 1929, 4, 385.

† It may be remarked that stratified films should, theoretically, be birefringent.

suitable for the study of the Haidinger interferences. The technique of observation described in an earlier page—namely, that of holding a smooth white illuminated screen containing a viewing aperture very close to the specimen—is particularly well suited for use with crystals. For, with the close approach

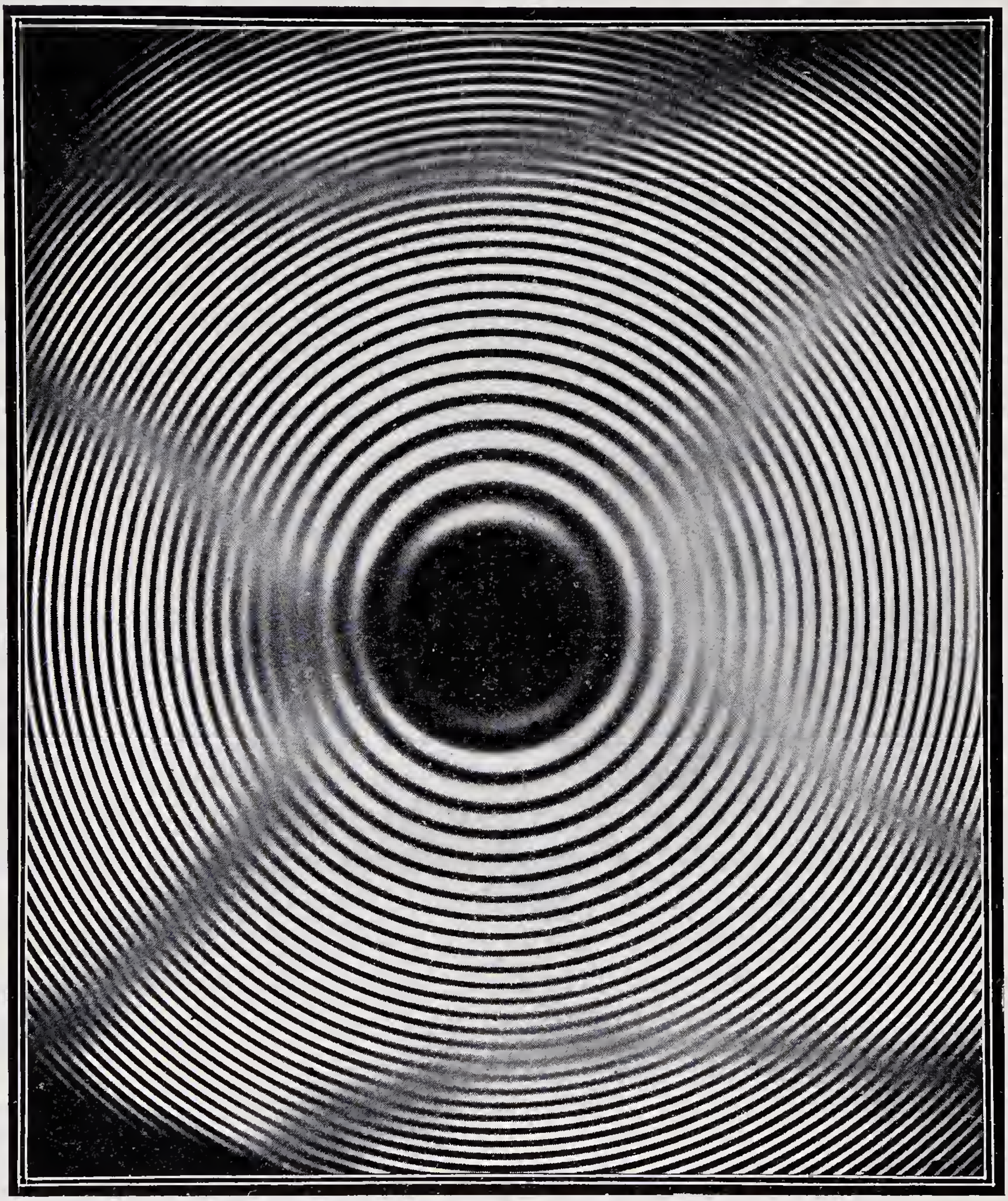

Fig. 11
Haidinger's Rings in Mica

of the observer's eye to the plate made possible in this way, an extended area of surface coupled with perfect uniformity of thickness becomes unnecessary, and it is possible to see and photograph a large number of rings satisfactorily even with small and not quite perfect specimens *by reflected light.* A photograph of Haidinger's rings obtained in this way with a sheet of mica and the 4358 A.U. radiations of the mercury arc is reproduced as Fig. 11. The rings as seen by *normal transmission* are, of course, weak. But they may be greatly improved in this respect by half-silvering the surfaces of the plate, in which case the bright rings also become much sharper. For viewing the rings as formed by *reflection,* however, such silvering is unnecessary. An alternative way of observing the interferences of crystalline plates is to hold the specimen obliquely against a monochromatic source and to view the light reflected by or transmitted through it. In this case, of course, only a small part of the interference field is seen at a given time. But we have the advantage that the fringes are then widely separated and the minima (or maxima as the case may be) are also much sharper than in the rings formed by reflection or transmission at normal incidence.

On an examination of Fig. 11, it will be noticed that the rings do not appear with the same clearness everywhere, their visibility being a minimum along four arcs of roughly hyperbolic form. The interferences exhibit dislocations where they cut these arcs, the dark rings on one side running into the bright rings on the other and *vice versa,* while a distinct doubling of the interferences can be noticed where they run nearly parallel to the arcs of minimum visibility. The form of these arcs of minimum bears an obvious resemblance to the isochromatic curves in the birefringence colours exhibited by a sheet of mica in the polariscope. Indeed, the resemblance is seen to be perfect if the isochromatic curves are viewed with a plate *twice as thick* as that used for the observation of the Haidinger's rings and the observations include a sufficiently wide range of angles. The curves of minimum visibility have then exactly the same shape as the interference figures in polarised light.*

From the facts stated, it is evident that we have, in fact, two sets of Haidinger's rings which overlap and which are seen most clearly where they are in coincidence, and least clearly where the bright rings of one system falls on the dark rings of the other and *vice versa.* That these two sets correspond to light polarised with its vibrations respectively along two perpendicular directions can be directly verified by looking at the ring system through a nicol; as this is rotated, the lines of minimum visibility disappear absolutely in four positions of the nicol at right angles to each other. The two systems therefore also correspond to the two different velocities of propagation of light through the crystal, and the directions in space in which they appear are hence given by the two formulæ

$$\delta_1 = 2dv_0/v_1 \cdot \cos r_1, \ \delta_2 = 2dv_0/v_2 \cdot \cos r_2$$

where v_0 is the wave-velocity in air, and v_1, v_2 are the wave-velocities in the crystal, while r_1 and r_2 are the corresponding angles of refraction of the light into the plate. The lines of minimum visibility of the interferences accordingly correspond to

$$\delta_1 - \delta_2 = 2d \cdot v_0 (\cos r_1/v_1 - \cos r_2/v_2) = (2p + 1)\lambda/2.$$

This is also the well-known formula giving the form of the isochromatic lines in convergent polarised light for a plate of thickness $2d$.

Since the wave-velocities v_1 and v_2 in a biaxial crystal depend on the direction of propagation, it is apparent that the rings cannot have a circular form as in the case of an isotropic plate. Their configurations can however be deduced from the expressions given above and the known form of the wave-velocity surface in the crystal. They are, in general, curves of the fourth degree whose form depends on the crystal and on the direction in which the plate is cut. In the particular case when the plate is normal to the plane containing the binormals, and provided the angular separation

* T. K. Chinmayanandam, *Proc. Roy. Soc.*, A, 1918, 95, 176.

of the latter is sufficiently great (as in the case of muscovite mica), the form of the curves in the vicinity of the normal to the plate can be shown to be approximately ellipses, their equations being

$$a^2y^2 + b^2x^2 = \text{constant, and } b^2x^2 + c^2y^2 = \text{constant},$$

where a, b, c are the three principal wave-velocities in the crystal. It is readily shown that the overlapping of two sets of rings of this form would give a set of hyperbolæ as the lines of minimum visibility.

The case of uniaxial crystals is also of interest. We may consider three typical examples, *viz.*, a plate normal to the optic axis, a plate parallel to it and a plate cut at an angle of 45°. The configuration of the "isochromatic" curves in these cases is well known. They are in the first case, a set of widely spaced circles, in the second case a family of rectangular hyperbolæ which are wide apart near the centre of the field and crowd together towards the margin, and in the third case a series of curved arcs running parallel and close to each other throughout the field. It follows that when unpolarised light is used, the Haidinger rings would be best seen with the first plate, less clearly with the second and should be scarcely visible in the third. Observations were made* with three plates of quartz five millimetres thick (figured by the firm of Hilger) and having the orientation stated, the rings being observed both by reflection and by transmission, in the latter case after half-silvering the plates. Polarisation of the incident light made a great improvement in the visibility of the rings as seen by reflection with the second plate; when it was rotated in its own plane, the rings were clearly seen in four positions and were very confused in four intermediate positions. The positions of the rings were also different for settings of the crystal at right angles to each other. The bifurcation of the rings could be clearly observed in the interference pattern as seen by transmitted light with the half-silvered plates, towards the margin of the field with the

* P. N. Ghosh, *Proc. Ind. Assoc. Cult. Sci.*, 1921, 7, 57.

first plate and even at the centre with the second. It was evident from these observations that while both sets of Haidinger rings in the first plate were circular, in the second plate one set was circular and the other was elliptic.

Colours of Stratified Media.—Several examples are known of substances exhibiting interesting optical effects ascribable to their possession of a periodic laminated structure. The optical behaviour of such substances includes a variety of phenomena, some of which lie outside the scope of the present lecture. We shall here consider only such of them as come under the category of the interferences of thin plates. Their essential features may be understood by considering the case of a medium made up of a succession of alternate strata of two different substances having a thickness d_1 and d_2 and refractive index μ_1 and μ_2 respectively. A beam of parallel light enters and tranverses such a medium, its angles of refraction into the alternate strata being r_1 and r_2 respectively. The reflections to which it gives rise at the boundaries of each layer would extinguish each other by interference

$$\text{if } 2\mu_1 d_1 \cos r_1 = n_1 \lambda$$
$$\text{or if } 2\mu_2 d_2 \cos r_2 = n_2 \lambda$$

as the case may be. On the other hand, if the condition

$$2\mu_1 d_1 \cos r_1 + 2\mu_2 d_2 \cos r_2 = n\lambda$$

is satisfied, the reflections at all the boundaries form two sequences in each of which there is complete agreement of phase. Accordingly, if this condition be satisfied, the advance of the incident wave through the medium results in the successive reflections reinforcing each other. We have then a strong wave reflected *backwards* which ultimately becomes as strong as the primary wave itself. *Per contra,* when the wave thus reflected travels backwards, it meets the successive boundaries in the reverse order, and gives rise to a second series of reflections which join up and build a strong wave travelling *forward.* This being in the same direction as the incident radiation, they interfere with the result that the amplitude of the primary wave progressively diminishes until it is finally extinguished. The net result is thus a *total reflection* in the backward direction of the incident light when it has traversed a sufficient depth of the medium.

We may ask at this stage, how many laminæ are required to give a sensibly perfect reflection? The answer depends on the strength of the resultant reflection from a single pair of strata in the medium. The greater the strength of the individual reflections, the more quickly would they add up so as to approach totality. It follows that the number of pairs of strata necessary to secure this result would be of the same order of magnitude as the reciprocal of the amplitude of the reflection from a single pair of strata. The incident wave would then penetrate into the medium to a depth not much greater than this number of strata. Accordingly, the rest of the medium is superfluous and may be removed without the totality of the reflection of this particular wave-length being sensibly affected.

The nature of the result to be expected if the relation

$$2\mu_1 d_1 \cos r_1 + 2\mu_2 d_2 \cos r_2 = n\lambda$$

is not exactly satisfied, clearly depends on the extent to which λ diverges from the value given by the relation. It is *prima facie* evident that the reflection would be sensibly total also for values of λ which differ from it, provided that the resulting disagreement of phase between the reflections from the first and last layers which sensibly contribute to its intensity is a sufficiently small fraction of the wave-length, say one-fourth or less. Accordingly, we infer that *the reflection would be sensibly total over a finite range of wave-lengths which is proportional to the reflecting power of a single pair of strata and is independent of the total number of strata present,* if this is sufficiently great. Outside the selected range of wave-lengths, the intensity of reflection must fall off with extreme rapidity. For, any failure of totality of reflection would involve a greater depth of penetration and thus introduce more reflections which would conspire to extinguish the effects of the earlier layers by interfering with them.

It is evident from the foregoing discussion that when white light is incident on a regularly stratified medium, we obtain a reflection which is more or less perfectly monochromatic, the range of wave-lengths included diminishing with the reflecting power of the individual laminations, a sufficiently large number of these being assumed to be present. The reflecting power of an individual pair of strata would be small and the reflection would therefore be highly monochromatic if the alternate strata are nearly equal refractive index or if one of them is of vanishingly small thickness. The number of laminations present influences the intensity and spectral character of the principal band of reflection only when the individual layers reflect so feebly that the radiation of the selected wave-lengths penetrates the entire depth of the medium.

It is evident also that weaker subsidiary maxima of reflection would accompany the principal band of reflection in the spectrum. For, the wave-lengths outside the range of total reflection would penetrate freely into the medium and the successive reflections would be all of comparable amplitude and of progressively altering phase. Hence, their resultant would not generally vanish but would remain finite, exhibiting oscillations of intensity which progressively diminish as we may move away from the principal band of reflection. The intensity and spread in the spectrum of these subsidiary maxima of reflection would be determined by the number of laminations present. They would be most noticeable when this number is small.

It should be remarked also that the *distribution of intensity of the subsidiary maxima of reflection may be unsymmetric with respect to the principal band of reflection.* For, this distribution would depend on the strength of the reflection by a single pair of strata, and the principal band may well fall in a region where the strength of such reflection alters rapidly with the wave-length. If, for instance, one of the layers is of very small thickness in comparison with the other, the principal *maximum* of reflection would fall in a region of the spectrum in the near vicinity of the *minimum* of intensity in the reflection by a single pair of strata. In such a case, it is evident that the subsidiary maxima would be weak on one side and strong on the other side of the principal maximum.

It is evident also that, in general, several orders of monochromatic reflection would be possible in the spectrum, depending upon the thickness of the stratifications and their refractive index. The relative intensities with which these orders appear would depend on the ratio of the thickness of the alternate strata and for particular values of this ratio, some of the orders of reflection would be missing. Why this should happen becomes clear if we express the periodic variation of refractive index along the normal to the stratifications as a Fourier series of harmonic components. Provided that the variations of refractive index are sufficiently small, each Fourier component thus derived can give rise only to one order of reflection which is observable when the wave-length of the incident light and the angle of its incidence satisfy the relation $2\mu\,\Delta \cos r = \lambda$, Δ being the spacing of the Fourier component. The absence of a particular Fourier component would thus involve the non-appearance of the corresponding order of reflection. This way of approaching the subject is of value as it enables us more generally to appreciate how the character of the stratifications determines the spectral distribution of intensity amongst the various orders of reflection. It is evident, in particular, why stratifications with a discontinuous distribution of refractive index favour the appearance of a large number of orders of reflection. It should be remarked, however, that the non-appearance of particular orders of reflection with certain types of discontinuous variation of refractive index can also be explained without the aid of the Fourier analysis, by a direct calculation of intensities.

The mathematical theory of the phenomena described above in general terms has been developed in a simple and elegant manner by G. N. Ramachandran,* and we proceed to give a sketch of it. As already indicated, we have, within the medium, two streams of energy, one travelling forwards and the other backwards. Each layer contributes to the backward stream of energy by reflecting part of the forward stream, and *vice versa*, while it reduces the strengths of these streams directly by transmission through it. Accordingly, if we denote the reflection and transmission coefficients of each layer by the complex numbers r and t respectively, then we have

$$R_s = r\,T_s + t\,R_{s+1}$$
$$T_s = r\,R_s + t\,T_{s-1}$$

where R_s and T_s represent the amplitudes of the backward and forward streams of energy at a point midway between the $(s-1)$th and the sth layers. Combining successive equations of the above type, we obtain

$$T_{s-1} = y\,T_s - T_{s+1} \text{ and } R_{s-1} = y\,R_s - R_{s+1},$$

where y stands for $(1 - r^2 + t^2)/t$. Assuming that there are only n laminations, we note that $R_{n+1} = 0$, from which it is readily deduced that

$$R_1 = f_n(y)\,R_n$$
$$T_1 = \left[\frac{1}{t} f_n(y) - f_{n-1}(y)\right] T_{n+1}$$

and
$$R_1 = \frac{r}{t} f_n(y)\,T_{n+1}$$

where $f_n(y)$ is the finite series $y^{n-1} - \dfrac{(n-2)}{\lfloor 1}\, y^{n-3} + \dfrac{(n-4)(n-3)}{\lfloor 2}\, y^{n-5} - \cdots$ to $(n-1)/2$ terms if n is odd, and to $(n-2)/2$ terms if n is even. Putting $y = 2\cosh\beta$, and $r/\sinh\beta = \sinh\alpha$, the series $f_n(y)$ sums up to the expression $\sinh n\beta/\sinh\beta$, and we get

$$\frac{R_1}{\sinh n\beta} = \frac{T_{n+1}}{\sinh\alpha} = \frac{T_1}{\sinh(\alpha + n\beta)}$$

where
$$\sinh^2\alpha = \frac{\{(r+t)^2 - 1\}\{(r-t)^2 - 1\}}{4r^2}$$

and
$$\sinh^2\beta = \frac{\{(r+t)^2 - 1\}\{(r-t)^2 - 1\}}{4t^2}.$$

* G. N. Ramachandran, *Proc. Ind. Acad. Sci.*, 1942, 16, 336.

These expressions are quite general, and involve no assumptions as to the nature of the reflecting layers. If now, we take the stratifications to be of the type considered in the preceding discussion, then by a simple application of the electromagnetic theory of light, it can be shown that r and t have the values

$$r = \frac{-\eta\,(e^{ik\delta_1} - 1)}{(e^{ik\delta_1} - \eta^2)\,e^{\frac{1}{2}ik\delta_2}}; \quad t = \frac{(1-\eta^2)\,e^{\frac{1}{2}ik\delta_1}}{(e^{ik\delta_1} - \eta^2)\,e^{\frac{1}{2}ik\delta_2}}$$

where η is the reflecting power of the boundaries of separation between the two media, $\delta_1 = 2\,\mu_1 d_1 \cos r_1$, $\delta_2 = 2\,\mu_2 d_2 \cos r_2$, and $k = 2\pi/\lambda$. From these, putting $\frac{1}{4}\,k(\delta_1 + \delta_2) = \phi$, and $(\delta_1 - \delta_2)/(\delta_1 + \delta_2) = c$, we get

$$\sinh^2 \alpha = (\cos^2\phi - \eta^2 \cos^2 c\phi)\,(\sin^2\phi - \eta^2 \sin^2 c\phi)/\eta^2 \sin^2 (1+c)\phi.$$

$$\sinh^2 \beta = -\,4\,(\cos^2 \phi - \eta^2 \cos^2 c\phi)\,(\sin^2 \phi - \eta^2 \sin^2 c\phi)/(1-\eta^2)^2.$$

The expressions for the reflection and transmission intensities can be discussed using the foregoing values of $\sinh \alpha$ and $\sinh \beta$, and the results so obtained confirm those indicated by the general discussion. It is found that the spectrum of the light reflected at any angle can be divided into two regions, *viz.*, those of the primary and the secondary maxima. These correspond to the regions where $\sinh \alpha$ is imaginary and $\sinh \beta$ is real, and *vice-versa,* respectively. The width of the former region, which determines the sharpness of the reflection, decreases as the number of laminations n is increased, but soon reaches a limiting value. The secondary maxima, however, steadily increase in number as n is increased.

To illustrate these points, the spectral distribution of intensity in the neighbourhood of the first order principal maximum has been drawn in Fig. 12 for 2, 10, 20, 50 and 100 plates. The reflecting power η is supposed to be 0·1, and the value of $c = 0{\cdot}9$. The dotted line marks the curve on which the secondary maxima lie, and it shows how the intensity of these is widely different on either side of the principal maximum.

Iridescence of Potassium Chlorate Crystals.—Potassium chlorate crystallises in the prismatic class of the monoclinic system, commonly taking the form of flat plates whose faces are parallel to two of the crystallographic axes and are inclined to the third.

Twins showing the characteristic re-entrant angles at the edges are not infrequent. The precise circumstances which result in the formation of crystals exhibiting vivid colours are rather

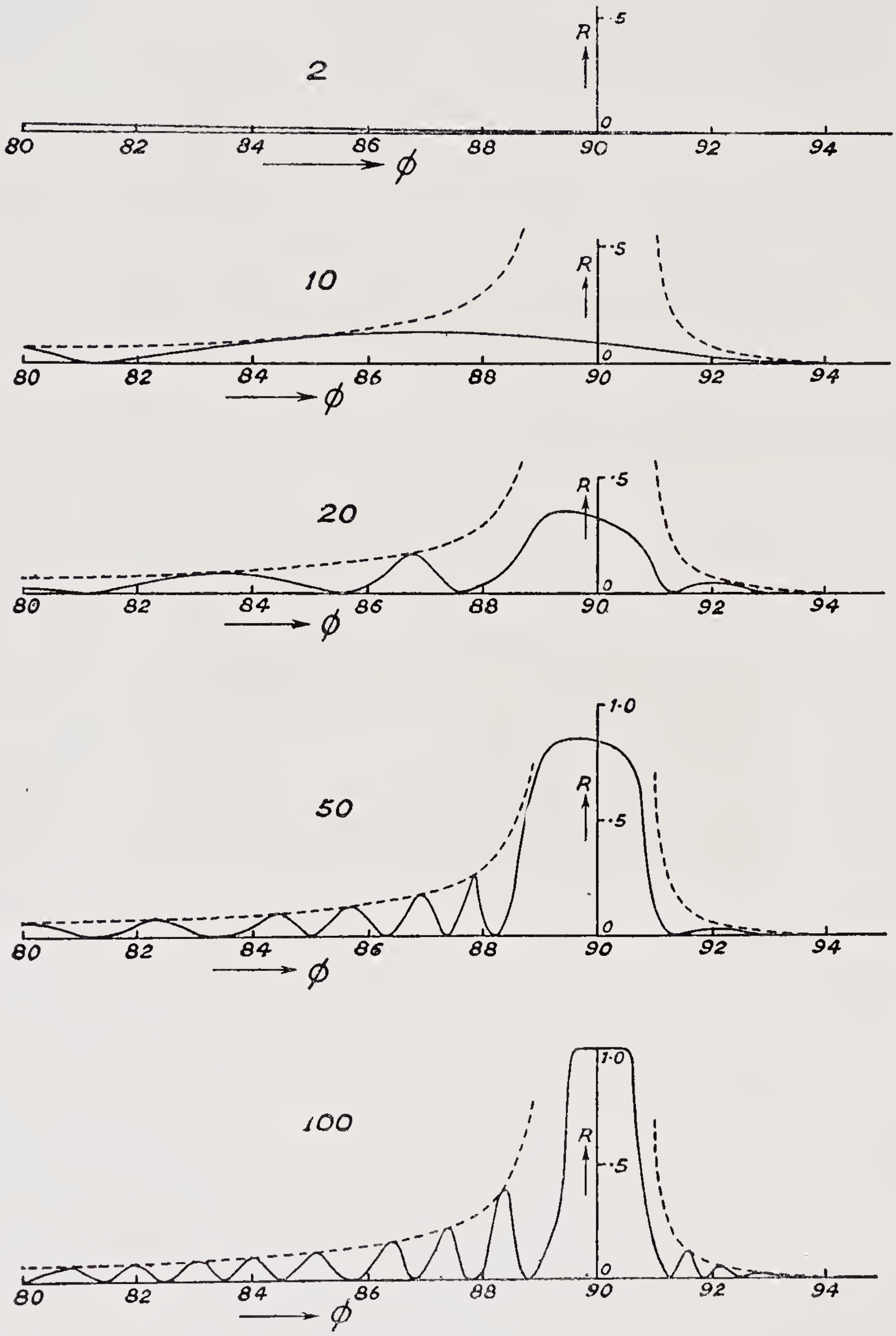

FIG. 12
Spectral Character of Reflection by a Regularly Stratified Medium

obscure. The seat of colour is usually a thin layer within the crystal parallel to its external faces and its power to reflect light appears to be the consequence of a repeated twinning within this layer. It is noteworthy that the coloured reflection vanishes when the plane of incidence of the light coincides with a plane of symmetry of the twinned crystal, while it is of maximum intensity when these two planes are perpendicular. Thus, when a crystal plate is held so as to reflect light obliquely and is turned round in its own plane, the colours alternately appear and disappear twice in each complete rotation. The intensity and colour of iridescence show wide variations. There are some crystals in which the reflections are so weak that they can only be observed with the flake mounted in Canada balsam between two glass prisms so as to eliminate all disturbing reflections. On the other hand, there are cases in which the coloured reflection is so intense that the crystal shines with an almost metallic lustre.

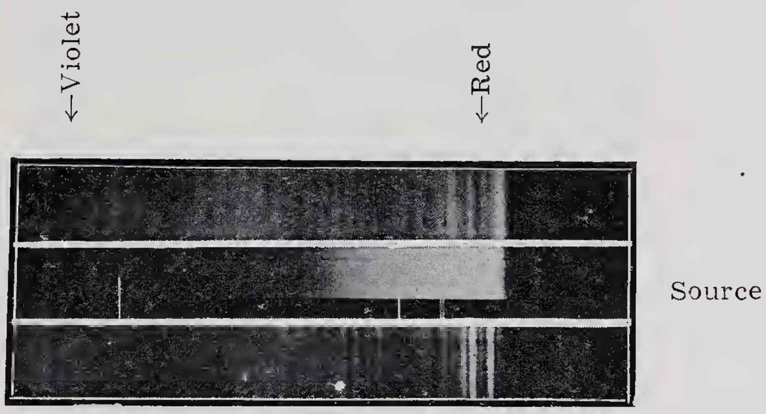

FIG. 13
Reflection Spectra of Potassium Chlorate Crystals
(Feeble Iridescence)

The spectral character of the reflected light is also very variable. In some cases, bands of varying widths are observed covering the whole of the visible spectrum. This is the case when the reflections are feeble and could, therefore, be ascribed to a few twinning planes only being present, possibly at considerable distances from each other and irregularly spaced (see Fig. 13). The character of the reflections shown by strongly iridescent crystals is more remarkable, being frequently found to consist entirely of a narrow region in the spectrum (Fig. 14). It is evident that the reflecting planes are then numerous and are arranged with remarkable regularity. In one such case,

L. A. Ramdas* observed no fewer than eight orders of reflection distributed in a regular sequence in the infra-red, visible and ultra-violet regions of the spectrum, each order of reflection being accompanied by a few subsidiary maxima on either side. The general similarity in the appearance of the different orders indicated that these subsidiary bands were of similar origin in each case.

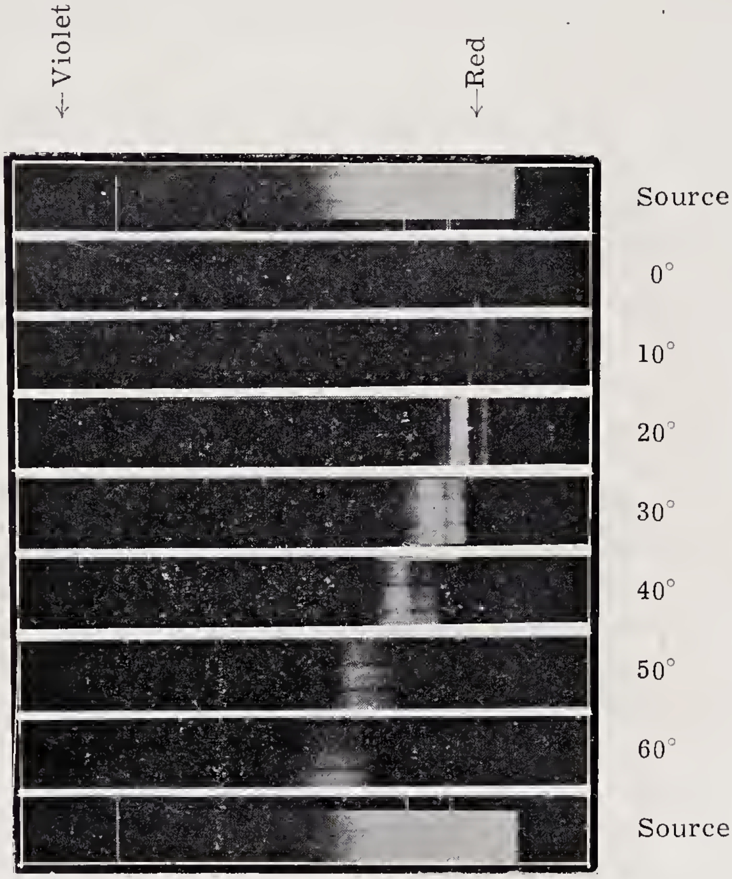

FIG. 14
Reflection Spectra of Potassium Chlorate Crystals
(Strong Iridescence and Varying Obliquity)

At strictly normal incidence, the twinning planes produce no optical effect, and there is then no hint of colour either by reflection or by transmission. On tilting the crystal even by a few degrees, the reflections appear, and their intensity increases rapidly as the incidence is made more oblique. The transmission colours are scarcely noticeable when the incidence is nearly

* L. A. Ramdas, *Proc. Ind. Assoc. Cult. Sci.*, 1923, 8, 231.

normal, but rapidly becomes richer at more oblique incidences. The spectra of the reflected and transmitted light are, of course, complementary, a sharp bright band in reflection corresponding to an equally sharp dark band in transmission. (Compare Figs. 14 and 15 which refer to the same crystal.) In the case of the strongly iridescent crystals, the selective reflections are practically total, and the corresponding extinctions in the transmitted light

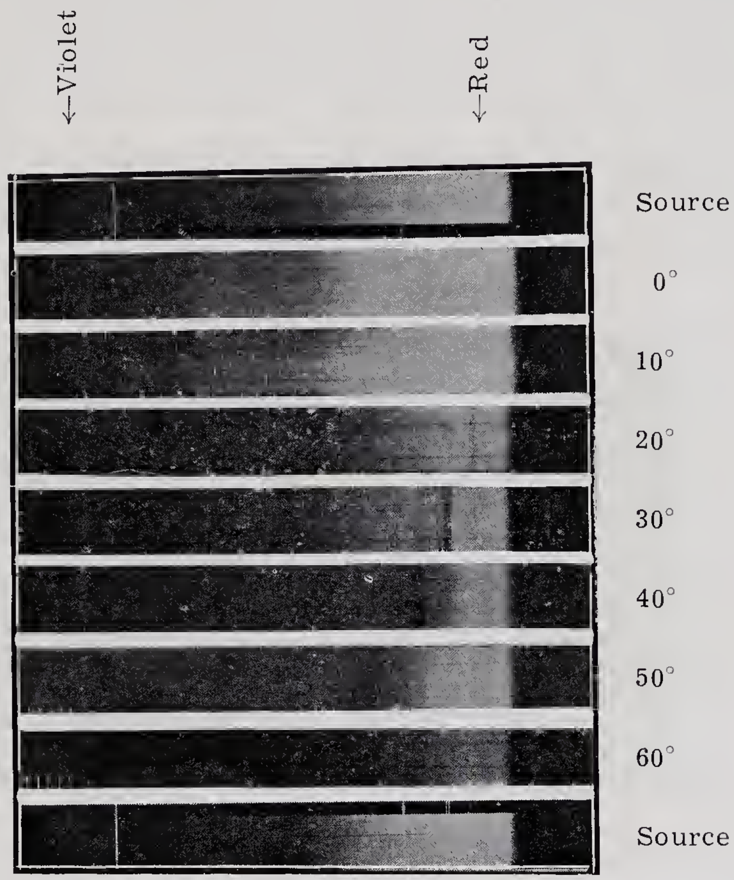

Fig. 15
Transmission Spectra of Potassium Chlorate Crystals
(Strong Iridescence and Varying Obliquity)

are therefore complete. The effect of varying the obliquity of incidence on the character of the spectra is very striking. The bands shift towards the violet end of the spectrum and at the same time rapidly widen out. The increased intensity of reflection and the enriched colour of the transmitted light are thus satisfactorily explained. With the particular crystal already mentioned, Ramdas found that the width $\Delta\lambda$ of the principal bands of reflection measured in Angstrom units decreased very markedly with increasing order of the reflection at any particular incidence. $\lambda/\Delta\lambda$ was not far from being the same for the different orders of reflection, but diminished rapidly with increasing obliquity of incidence, being about 150 near normal incidence, about 75 at 23° and 40 at 50° away from the normal.

The increasing width of the spectral bands of selective reflection at oblique incidences is evidently the result of the increased reflecting power of an individual lamina at such incidences, and is thus an illustration of the general theory of reflection by a stratified medium. A similar result may be brought about by keeping the angle of incidence constant, but rotating the crystal in its own plane. We have already remarked that when this is done, the

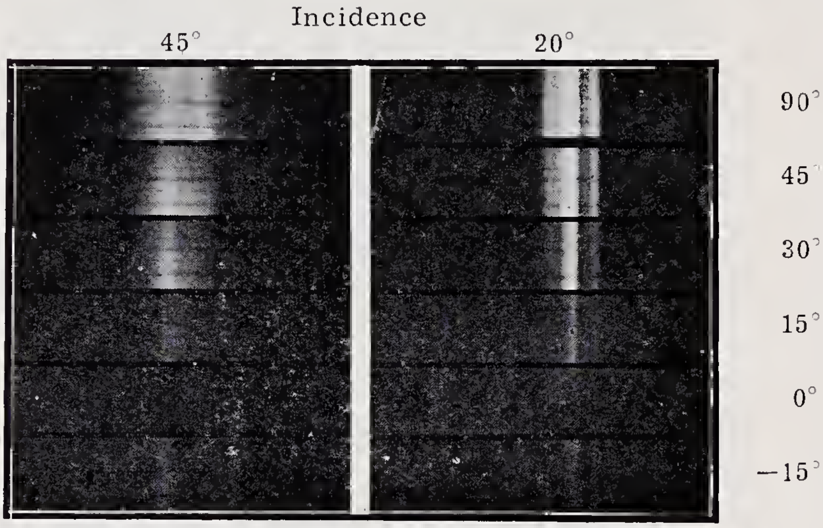

Fig. 16
Reflection Spectra of Potassium Chlorate Crystals
(Effect of Varying Azimuth of Reflection)

reflection vanishes twice in each complete rotation. Accordingly, we should expect that the width of the spectral bands of selective reflection should oscillate as the crystal is rotated, being least when the reflection is nearly extinguished and greatest when it is most intense. Further, these changes should be the more pronounced the greater the angle of incidence on the crystal. These anticipations from theory are completely confirmed in experiment.* (See Fig. 16.)

* Unpublished observations by V. S. Rajagopalan and the present writer.

A remarkable prediction from theory is that the direction of vibration of the light vector is turned through a right angle in the act of reflection from a twinning plane.* In other words, if the incident light is polarised in the plane of incidence, the coloured reflections should be polarised in the perpendicular plane, and *vice versa.* The late Lord Rayleigh to whom the explanation of the iridescence of potassium chlorate crystals is due, endeavoured to verify this prediction but found he could do so only in the particular case when the incidence of light on the crystal is nearly normal. As pointed out by him, the difficulty in observing the effect at more oblique incidences is the depolarisation of the light in its passage though the crystal before and after reflection at the twinning planes. This difficulty, however, disappears if the coloured layer be sufficiently close to the external surface of the crystal. Observations can then be made even at oblique incidences.‡ One polaroid may be used to polarise the incident light, while the reflected light is viewed through a second polaroid and a direct vision spectroscope, the latter serving to eliminate the disturbing reflection at the external surface of the crystal. It is observed, in agreement with the theoretical prediction, that *the spectral bands of selective reflection are strong when the two polaroids are crossed and very weak when they are parallel.* A rotation of one polaroid alone when the other is absent makes no difference in the intensity of the coloured reflection. This is also in accordance with the theory.

It may be remarked that the existence of the twinned strata in potassium chlorate is no mere hypothesis, as they can be made visible by suitable sectioning, coupled with the use of polarised light for microscopic observation.† Viewing a crystal plate directly under the polarising microscope also discloses the difference between a twinned and an untwinned specimen. X-ray examination of twinned and iridescent crystals shows a doubling of certain groups of Laue spots which appear single in the untwinned crystals.§ A similar behaviour is shown by the

* Rayleigh I, *Scientific Papers,* 3, 201.
‡ Unpublished observation by the present writer.
† Rayleigh II, *Proc. Roy. Soc.,* A, 1923, 102, 668.
§ S. C. Sirkar, *Ind. Jour. Phys.,* 1930, 5, 337.

crystals in which twinning has been developed by heating up to nearly the fusion temperature and subsequent cooling. The question why twinning occurs with such facility and with such remarkable regularity in many cases in potassium chlorate is of considerable interest and is worthy of fuller elucidation. It is presumably connected with some special feature in the crystal structure of the substance.

Structure and Colours of Opal.—Precious opal exhibits a striking play of colour. The finest specimens give brilliant monochromatic reflections over large areas, the colours ranging over the whole spectrum and altering with the angle of incidence of the light. Some specimens exhibit numerous small glittering spangles of colour, and others again an almost continuous sheen of iridescence. Some very beautiful and valuable opals are grey, blue or black in colour, the iridescence showing up by reflection against the dark background thus provided. Opals of a lighter tint are fairly transparent and in transmitted light exhibit hues approximately complementary to the colour of the reflected light. Opals also usually show a bluish-white opalescence overlying the reflected colours, and if such opalescence is strong, the colour seen by transmitted light tends to a honey-yellow, the complementary tints then being less conspicuous.

In examining the spectra of the reflected and transmitted light, it is important to fix attention upon a limited area of the opal showing a definite colour. The reflected light then appears as a narrow region in the spectrum, the width of which at normal incidence may be as little as 50 Angstrom units or less. Corresponding to the bright band in the reflection, a black band appears in the transmitted light, indicating that the reflection is total (Fig. 17). On tilting the reflecting surface away from normal incidence, the bands shift towards the violet, broadening as they move. While these features are analogous to those observed with potassium chlorate, in almost every other respect the opal colours behave differently. They do not vanish when the incidence is normal, nor do they vary in intensity or spectral character with the azimuth of incidence. They are also polarised in the normal way, as may be shown by immersing the opal in carbon disulphide, thereby enabling the angle of incidence to be increased up to and beyond the angle required for complete polarisation. If polarised light is incident on the opal and the iridescent reflections are observed through a second polariser, they may be extinguished by holding the two polarisers in the crossed position, while in the parallel position the iridescence is seen with maximum intensity.

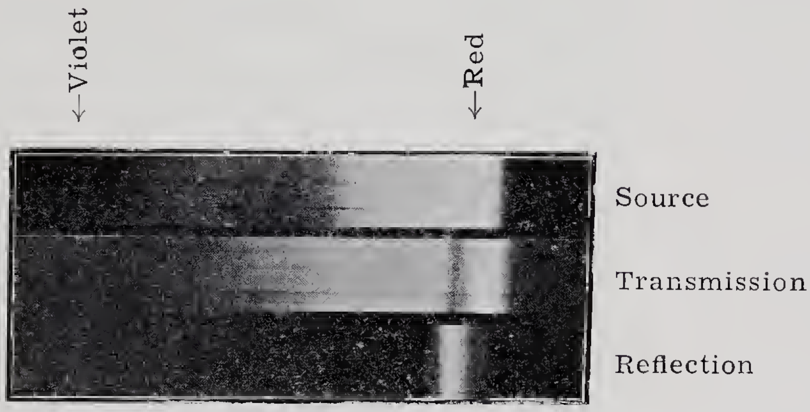

FIG. 17
Reflection and Transmission Spectra of Opal

Examination of opal under the microscope reveals that the material possesses a lamellar structure of a remarkable character, there being actually three sets of interpenetrating planes of lamination geometrically related to each other in a manner recalling the rhombohedral cleavages of a crystal of calcite. Each set of laminations is capable of giving a monochromatic reflection of which the wave-length varies with the angle of incidence according to the usual formula. If, in a particular area, all three sets of laminations are exposed, they should be capable of giving reflections of a wave-length determined by the respective angles of incidence of the light upon them. But it is obvious that if the incident light be parallel, it would not be geometrically possible to observe the three reflections simultaneously. Normally therefore, if one set of laminations appears bright by reflection, the two others would, in general, appear dark. If, however, light be allowed to fall on the specimen from different directions at the same time, it should be possible to see the reflections from two or even all the three sets of laminations simultaneously. They would then be visible as sharply divided areas exhibiting different colours in close juxtaposition. The effect is readily observed and is a striking demonstration of the existence of the three sets of laminations.

Figs. 18 and 19 are photographs of the same area on a piece of Australian opal, nothing being varied except the direction of

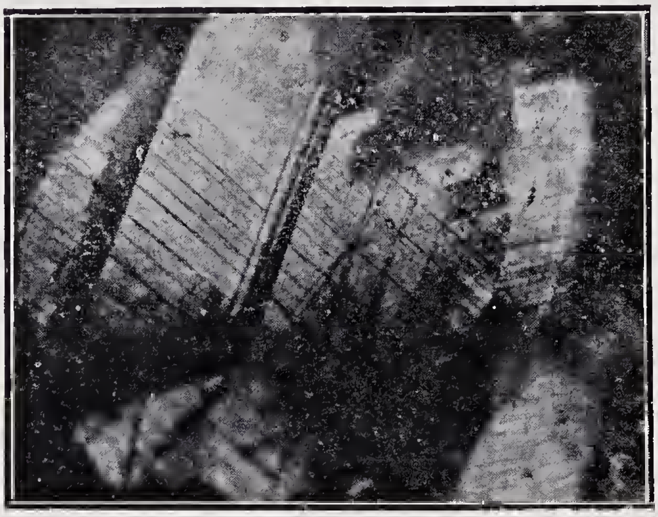

FIG. 18
Laminated Structure of Opal

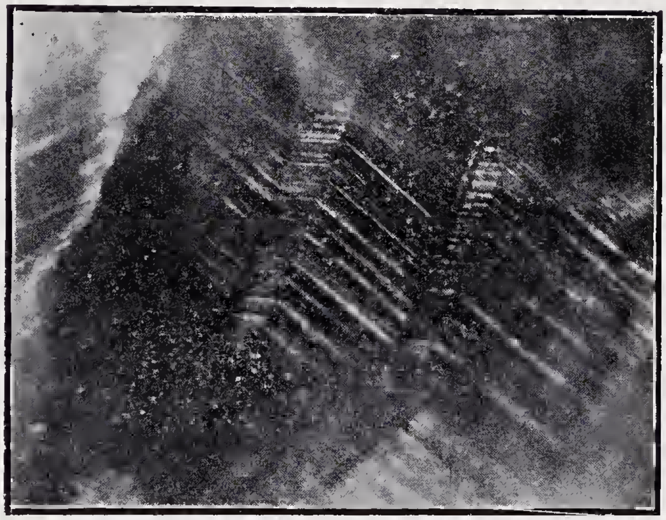

FIG. 19
Laminated Structure of Opal

illumination. The surface of the specimen was roughly parallel to the exposure of one of the three sets of laminations exhibiting a bright green iridescence. This appears as an extended luminous area in Fig. 18. The second set of laminations presented exposed surfaces of lesser area appearing as transverse bands and lines cutting across the main luminous area. As seen visually under the microscope, these exhibited a violet reflection. The third set of laminations appears as dark lines crossing the second set obliquely in Fig. 18. In Fig. 19 the illumination was so arranged that this third set of laminations appeared as bright yellow lines running through the whole field, the other two sets of laminations being perfectly dark. Effects of this kind may be conveniently studied by placing the specimen on the stage of a microscope, illuminating it from above in any desired direction and rotating the stage.

As the spacing of the stratifications is of the order of the wave-length of light, we can scarcely hope to be able to observe them directly under the microscope except in specially favourable circumstances. Their physical nature and origin is, as yet, an unsolved problem, though numerous studies by X-ray and other methods have been reported in the literature. The spectral character of the reflections indicates that these stratifications are numerous and regular, and also that the reflective power of an individual lamina is not great, thus definitely excluding such crude hypotheses as for instance, the presence of cracks containing air. It appears probable that the material of the stratifications differs little in refractive index from that of the opal substance, or, alternatively, that the reflecting layers are thin even in comparison with the spacing of the laminations. Precise measurements of the polarising angle of reflection may assist in reaching a decision as to the nature of the material present.

Colours of Mother-of-Pearl.—A nacreous layer with a characteristic lustre and iridescence is present in the shells of a great many mollusca. The nature of the material is, however, far from being identical in the different classes of mollusca, *e.g.*, the Bivalves, the Gastropods and the Cephalopods. This is indeed evident from the striking variations in the general appearance of the nacre as well as of its density and other physical properties. Closer examination reveals remarkable variations in the structure and optical properties of nacre, not only of these great classes of mollusca but also of individual genera and species. These facts as well as the ready availability of the material, and the ease with which it can be worked and polished, make mother-of-pearl a substance of considerable interest to the student of optics. Large-sized shells of *M. margaratifera, Turbo, Trochus, Haliotis* and *Nautilus* are easily obtained, and when their nacreous layer is exposed and polished, they make striking exhibits. The iridescence of a large shell of *Turbo* thus prepared may be effectively displayed by placing an electric bulb inside it. The soft and glowing colours of the light which then diffuses out of the shell make an impressive demonstration of its optical properties.

A convenient way of examining the colours of mother-of-pearl is to cut out a piece of the shell parallel to its surface and after grinding it down to a suitable small thickness to polish its surfaces and mount it in Canada balsam between two cover slips of glass. To illustrate the influence of the thickness of the piece on the spectral character of the effects, the material may be worked into the shape of a wedge, not very thick at one end and tapering off to extreme thinness at the other. The transmitted light may then be examined by placing the mounted specimen right up against the slit of a pocket spectroscope. The transmission colour as visually observed is very weak at the thin end and becomes more vivid with increasing thickness. The spectroscope indicates that this effect is due to the spectral band of extinction being very narrow at the thin end and widening out with increasing thickness. This widening is evidently the result of small variations in the spacing of the laminations and illustrates the optical principle that a lack of perfect regularity in the stratifications may actually improve the intensity of iridescence.

Mother-of-pearl consists, in the main, of calcium carbonate which, remarkably enough, is present in the optically biaxial form of aragonite together with varying amounts of an organic substance known as conchin which serves to hold the substance together. The laminations characteristic of nacre are, in fact, made up of successive layers of aragonite in the form of very thin crystalline plates which are held together by the cementing material, there being an immense number of such layers, running roughly parallel to the natural surface of the shell. Examination of thin sections under the polarising microscope shows clearly that the aragonite crystallites all lie with their c-axes more or less exactly normal to the general direction of the laminations. The c-axis is the direction of vibration for which the refractive index of aragonite is least (1·530), while the refractive indices corresponding to the other axes are larger and nearly equal (1·680 and 1·685). For directions of incidence of the light not far from the normal, the effective refractive index of the crystalline plates is, therefore, in the neighbourhood of 1·68. The refractive index of conchin is probably about the same as that of solid gelatin (1·53), and the difference between this and the index of aragonite (1·68) is fairly large. Hence, if the thickness of the conchin films were at all comparable with those of the aragonite layers, the reflecting power of the individual laminæ would be so large that the reflections by a large number of them would not exhibit any very marked spectral selectivity. A consideration of the relative densities of aragonite, mother-of-pearl and conchin, however, indicates that the conchin layers should be very thin in comparison with the aragonite crystals, and this conclusion is independently supported by a microscopic examination of the material. It follows that the reflecting power of an individual lamination should be small, and the fact that mother-of-pearl gives monochromatic reflections and extinctions in much the same way as potassium chlorate and opal thereby becomes intelligible. It is also evident that the relative thickness of the conchin and aragonite layers in different varieties of nacre should influence its optical properties quite as much or even more than the variation in the spacing of the laminations. This view is supported by spectroscopic studies which reveal that the greater vividness of colour exhibited by the mother-of-pearl from certain mollusca, *e.g., Haliotidæ,* is largely a matter of the greater width of the spectral bands of extinction and transmission.

The angle at which the laminations meet the surface of the shell varies but is usually small; it may be adjusted to any desired value by suitably cutting and polishing the material. Unless the polishing has been unduly prolonged, the structure of nacre becomes evident under the microscope when its surface is illuminated in such manner that the direct reflection does not enter the field of view. Numerous sharp bright lines on a dark background are then seen (Fig. 20); these are the intersections of the

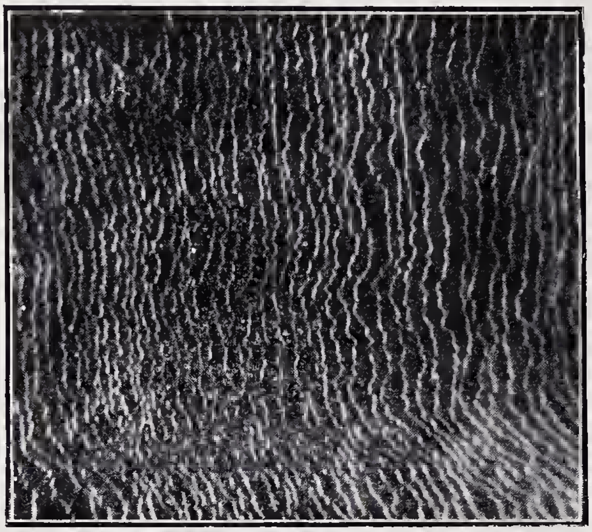

FIG. 20
Surface of Mother-of-Pearl under Dark-Ground Illumination

conchin layers with the surface of the shell, and their configuration depends upon the curvature of the intersecting surfaces and the angle at which they meet. The sharpness of the lines and the character of the optical effects observed makes it evident that the conchin layers are excessively thin compared with the aragonite crystallites which they separate.

A surface structure such as that illustrated in Fig. 20 necessarily gives rise to diffraction effects when a pencil of light falls upon it and reflected by it. Fig. 21 (A) exhibits the group of diffraction spectra given by a moderately well-polished surface and received on a screen, while Fig. 21 (B) is the effect observed when the surface is covered by a little Canada balsam and a glass cover-slip. It will be noticed that the diffraction spectra seen in Fig. 21 (A) have all vanished in Fig. 21 (B), except the third order on the left which persists with undiminished intensity and in an unaltered position. This is evidently the monochromatic reflection by the internal laminations.* As we shall see in a later

* C. V. Raman, *Proc. Ind. Acad. Sci.*, 1935, 1, 574.

lecture, it is a simple consequence of diffraction theory that the directions in which the spectrum of a particular order and wave-length and the selective internal reflection of the same order and wave-length appear, coincide. This fact enables us readily to

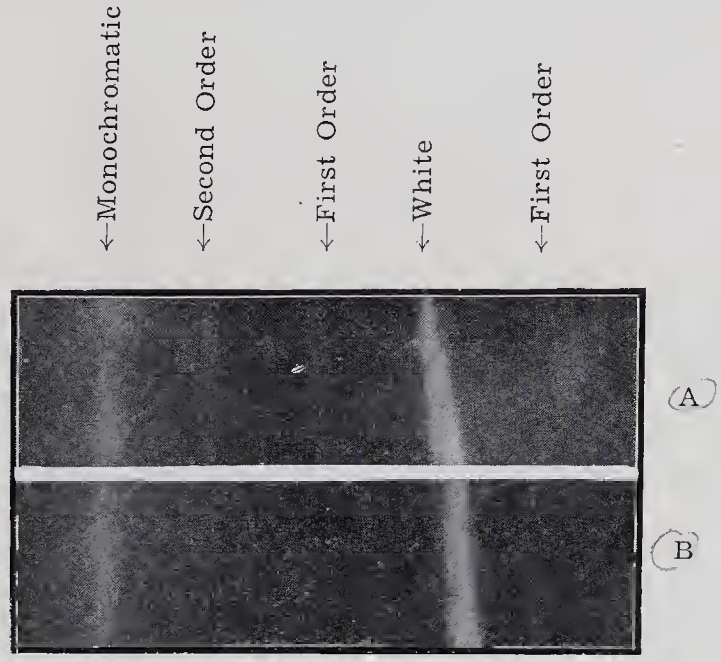

FIG. 21
Reflection and Diffraction by the Surface of Mother-of-Pearl

evaluate the spacing of the laminations optically and to connect their separation as seen on the surface with that directly observed (though with difficulty) in transverse sections under the microscope.

Mother-of-pearl is not only a stratified medium but also a heterogeneous one. This results in a diffusion of light which increases with the thickness of the material and exhibits itself in several ways. While a part of the incident light is selectively reflected, the rest is diffused backwards, forwards and laterally, while the regularly transmitted light progressively suffers extinction. The diffusion backwards, *i.e.*, towards the source of light, results in mother-of-pearl exhibiting a body-colour which is *complementary* to the iridescence. The effect is conspicuously seen when the material is viewed in directions in which the iridescence is not visible. Indeed, the body-colour is also present superposed on the iridescence, thereby diluting its spectral purity. How great such dilution is can best be realised by viewing the shell under the light of the open sky. The iridescence and body-colour then completely overlap, and the shell appears dead-white, even though under directed illumination it may be strongly iridescent. In certain cases, however, as for instance, some of the Californian Haliotidæ, the iridescence is very striking even in diffuse light. It would appear that in such shells the body-colour is suppressed by the presence of a strongly absorbing material in the conchin layers.

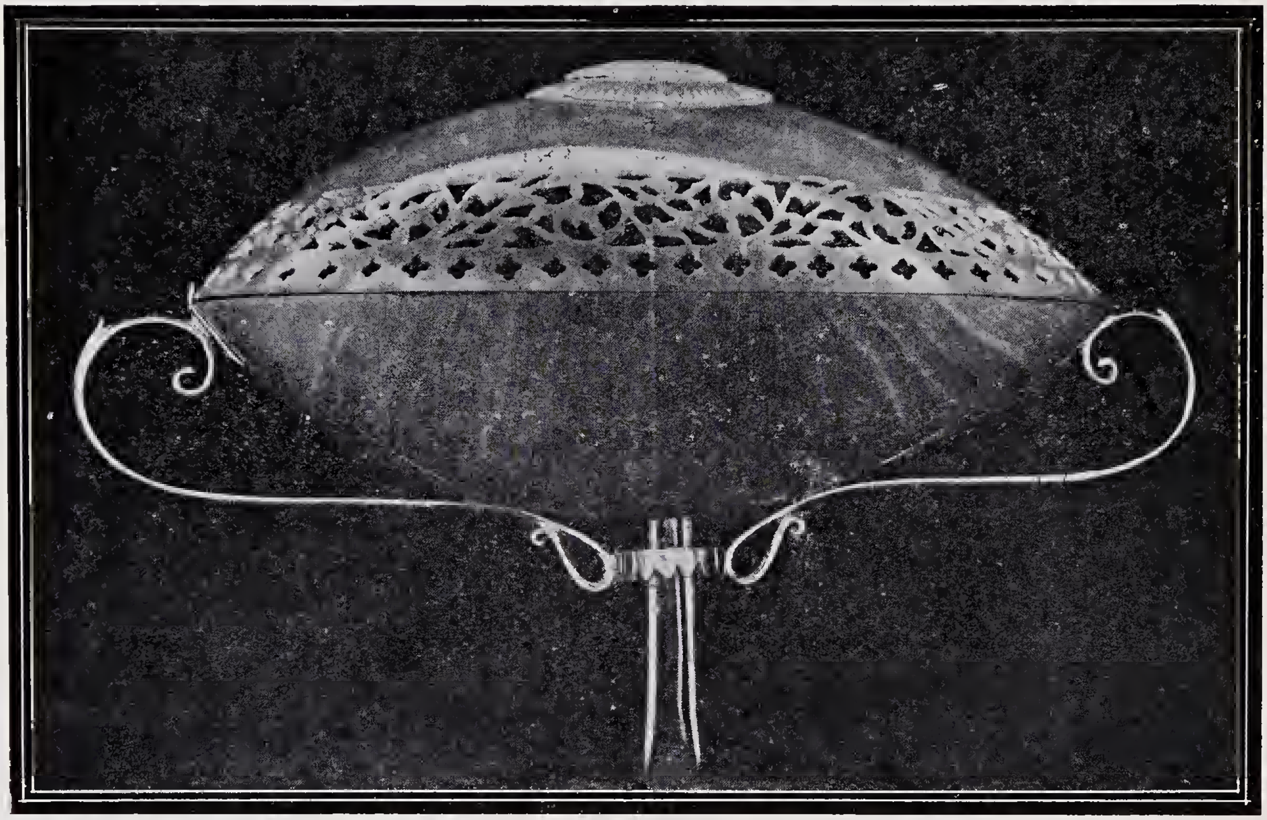

FIG. 22
Bands of Body-Colour in a Polished Shell of *Turbo*

The diffusion of light in nacre is, at least in part, due to the aragonite crystallites which give the substance a granular structure. This may be demonstrated by viewing a distant source of light through a thin piece of mother-of-pearl polished and mounted in Canada balsam between glass cover-slips and held normally in front of the eye. A diffusion halo is then observed surrounding the source of light, its form being radically different with the mother-of-pearl from the three classes of mollusca, namely the Bivalves, the Gastropods and the Cephalopods.* The haloes differ in detail while retaining the general

Gastropods (*Turbo*)

Bivalves (*M. margaratifera*)

Cephalopods (*Nautilus*)

Fig. 23

Diffusion Haloes of Mother-of-Pearl

features of the class for the different individual genera and species. Their configuration is determined by the size, shape, orientation

* C. V. Raman, *Proc. Ind. Acad. Sci.*, 1935, 1, 859.

and spacing of the aragonite crystallites and is illustrated for three typical cases in Fig. 23. The conclusions regarding the structure of mother-of-pearl in the three classes of mollusca reached from a study of the diffusion haloes are independently confirmed by various other methods, *viz.*, by a microscopic study of thin sections,* by their X-ray diffraction patterns$ and by observations of the birefringence,* the magnetic anisotropy‡ and the elastic behaviour of nacre§ from the different sources. We shall not enter into further details here, as this would take us beyond the limits of our present subject.

A lamination spacing of about $0 \cdot 5\,\mu$ may be taken as roughly representative of the strongly iridescent varieties of mother-of-pearl; this would give a third-order reflection in the brightest part of the visible spectrum, the second order being in the near infra-red, while the fourth order would be at the violet end of the spectrum. A tenth of a millimeter would accordingly be sufficient to include about 200 laminations, and if these were all uniform and equally effective, the bands of reflection and extinction would have a width of only 10 Angstroms, while at the same time, the effect of diffusion would be minimised. The advantage of using a relatively thin piece of the material for spectroscopic observations is thus apparent. With selected specimens and holding the slit of the spectroscope parallel to the contour lines of colour, reflection and extinction bands fully as sharp and straight as those observed with potassium chlorate crystals and with opal may be seen with mother-of-pearl.

The extinction of the transmitted light due to diffusion in traversing the nacre varies greatly with the class of mollusc. The mother-of-pearl from *Trochus*, for instance, is remarkably opaque even in thin layers, while the shells of *Turbo* and *M. margaratifera* are much more transparent than those of *Haliotis* or *Nautilus*. As is generally the case with turbid media, the extinction coefficient increases rapidly as the wave-length is diminished, becoming very great in the violet and ultra-violet

* V. S. Rajagopalan. *Proc. Ind. Acad. Sci.*, 1936, 3, 572.
$ S. Ramaswamy, *Ibid.*, 1935, 1, 871; and 1935, 2, 345.
‡ P. Nilakantan, *Ibid.*, 1936, 2, 621.
§ P. S. Srinivasan, *Ibid.*, 1937, 5, 463.

regions of the spectrum. The colour of the transmitted light which at first is complementary to the selective reflection, assumes a reddish tinge and ultimately becomes a deep red with increasing thickness of the plate. One curious result of this is that specimens which selectively reflect at the red end of the spectrum are more opaque than those which give green or blue reflections.

Fig. 24

Shell of *Nautilus* photographed as a Translucency

With thicknesses greater than a few tenths of a millimetre, light does not penetrate but only diffuses through; the colour of the diffusing light at first is complementary to the selective reflection only for moderate thicknesses and assumes a reddish tinge with increasing thickness. One remarkable property characteristic of nacre is that light can diffuse *in a direction parallel to the laminations to a distance of some centimetres,* while the diffusion normal to the laminations is limited to a few millimetres at the utmost. This effect is illustrated in Fig. 25 in which an extended area of the nacre appears luminous though only a small region at its centre was illuminated.*

* C. V. Raman, *Proc. Ind. Acad. Sci.,* 1935, 1, 859.

This effect illustrated in Fig. 25 is no doubt a consequence of the special structure of nacre. The shape and size of the aragonite crystallites vary with the class of mollusc, the smallest dimension being from 2 to 8 μ as shown by the angular diameter of the diffusion haloes, while their thickness is of the order $0{\cdot}5\,\mu$. The number of reflecting boundaries traversed per unit path of the light is thus many times greater transverse to the laminations

Fig. 25

Diffusion of Light in *Margaratifera* Parallel to Laminations

than parallel to them. The difference in refractive index between the aragonite and the conchin also effectively vanishes for light travelling parallel to the laminations with its electric vector normal to them. These features conspire to enable the light to penetrate to great depths along the aragonite layers.

Iridescence of Decomposed Glass.—Ancient glassware excavated from archæological sites is often distinguished by a beautiful iridescence. It is frequently possible to detach iridescent flakes from such glass, exhibiting rich colours by transmitted light complementary to those seen by reflection. The laminar structure of decomposed glass becomes evident on examining the edges of the flakes under the microscope. It is noticed also that the flakes are often not plane, but consist of shallow cups like watch-glasses fitting together perfectly and dividing the area into a large number of polygons. These exhibit rich colour which is uniform over a large area but differs slightly at the margin and centre of each cup, owing to the difference in obliquity of observation. The beautiful regularity in curvature and the sharpness of the polygonal edges are evident in Fig. 26 which reproduces the interference rings in monochromatic light seen by transmission between the upper surface of the flake and a sheet of mica laid above it, and viewed under a microscope.*

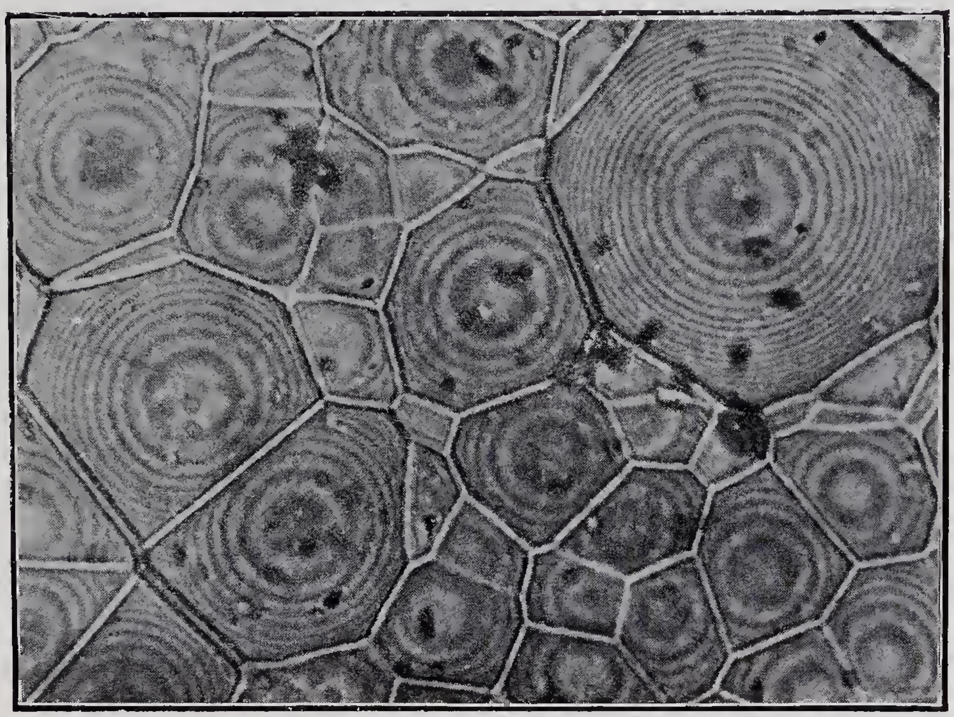

Fig. 26
Polygonal Sub-divisions in Iridescent Glass

Deeper cavities are sometimes seen which may be spherical or ellipsoidal in shape. Seen between crossed micols in the polarising microscope, such cavities exhibit a dark cross (Fig. 27) intersected by rings of colour due to the varying obliquity of the surface.

From the fact that the laminations in an iridescent flake adhere pretty firmly to each other, it is evident that they are *ordinarily in optical contact and are not separated by films of air.* When the flakes are viewed under the microscope in monochromatic light, the areas of colour appear uniform in intensity,

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* C. V. Raman and V. S. Rajagopalan, *Proc. Ind. Acad. Sci.*, 1940, 11, 469.

but in those cases where a film of air has entered and broken optical continuity, it reveals itself immediately by the appearance of bright and sharply defined interference fringes on a dark

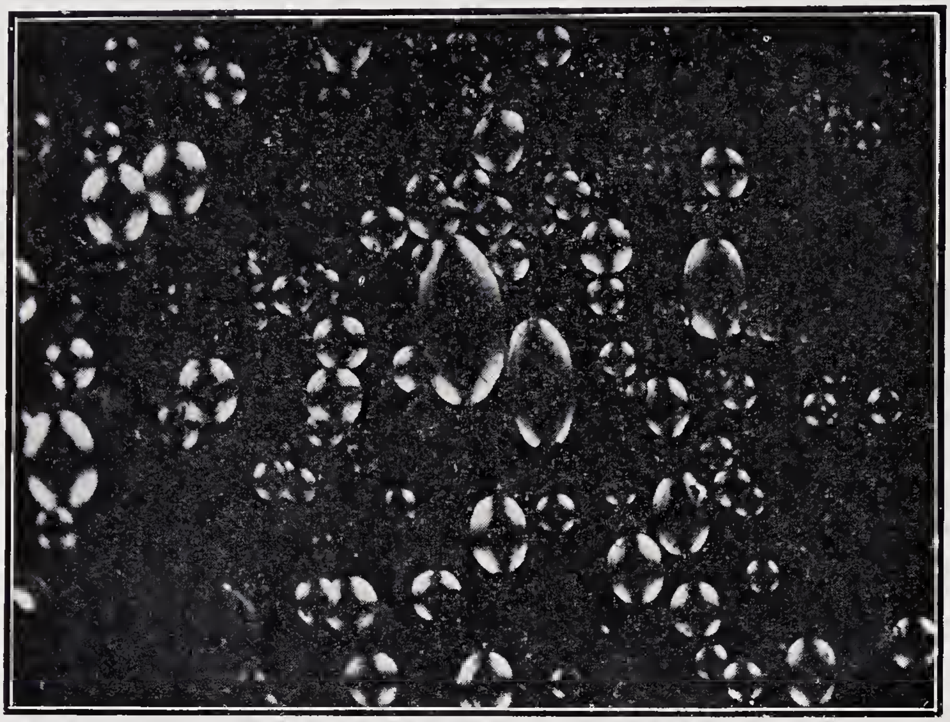

FIG. 27
Deep Cavities in Iridescent Glass under Polarised Light

ground (Fig. 28). Such fringes move about when the flake is lightly pressed and recover their shape when the pressure is removed. The presence of films of air also disturbs the uniformity of colour of the flakes as seen in white light under the microscope.

An insight into the nature of the laminations is obtained on immersing a flake in a cell of liquid and gradually varying the refractive index of the latter. The transmission colour disappears immediately, but the reflection colour continues to be visible though weakened, provided the refractive index of the liquid is either lower or higher than of the glass ($\mu = 1{\cdot}46$). It is noticed, however, that the colour of the flakes when immersed in carbon disulphide ($\mu = 1{\cdot}63$) is decidedly different from the colour when immersed in water ($\mu = 1{\cdot}33$). These observations indicate that the decomposed flake, though optically a continuous medium, has an open or porous texture into which liquid can penetrate, the distribution and size of the cavities varying in such manner as to give the material a laminated structure. A striking proof of this is given by the appearance of the flake when it is removed from the immersion liquid and allowed to dry. When still saturated

FIG. 28
Fringes due to Intruding Air Films

with liquid, it appears quite colourless. But immediately the drying commences, it becomes almost perfectly opaque and transmits no light, while it is silvery white as seen by reflection. The opacity then slowly clears up, the flake exhibiting the usual colours when completely dry. These observations indicate that in drying, the liquid withdraws first from the largest cavities, while it continues to fill the finer pores; as a consequence, the optical stratifications are actually intensified in the first stage of drying and their reflecting power is enhanced as compared either with the perfectly dry or with the completely saturated flake. The porous structure of ancient glass is also shown by placing a small flake covered with a piece of cellophane on the stage of the microscope, and then pressing or rolling a blunt steel point firmly on its surface. The colour is then observed practically to disappear from the area of pressure and does not recover on its removal.

It would seem that the laminations responsible for the colour are tolerably regular. This is indicated by a spectroscopic examination which shows that the flakes may be roughly classified in two divisions. Flakes of the first kind appear red or orange red by reflected light and blue or bluish-green by transmission; while flakes of the second kind show the complementary colours in each case. Typical spectra of the two groups are reproduced in Figs. 29 (A) and 29 (B). Spectra of the first kind are roughly

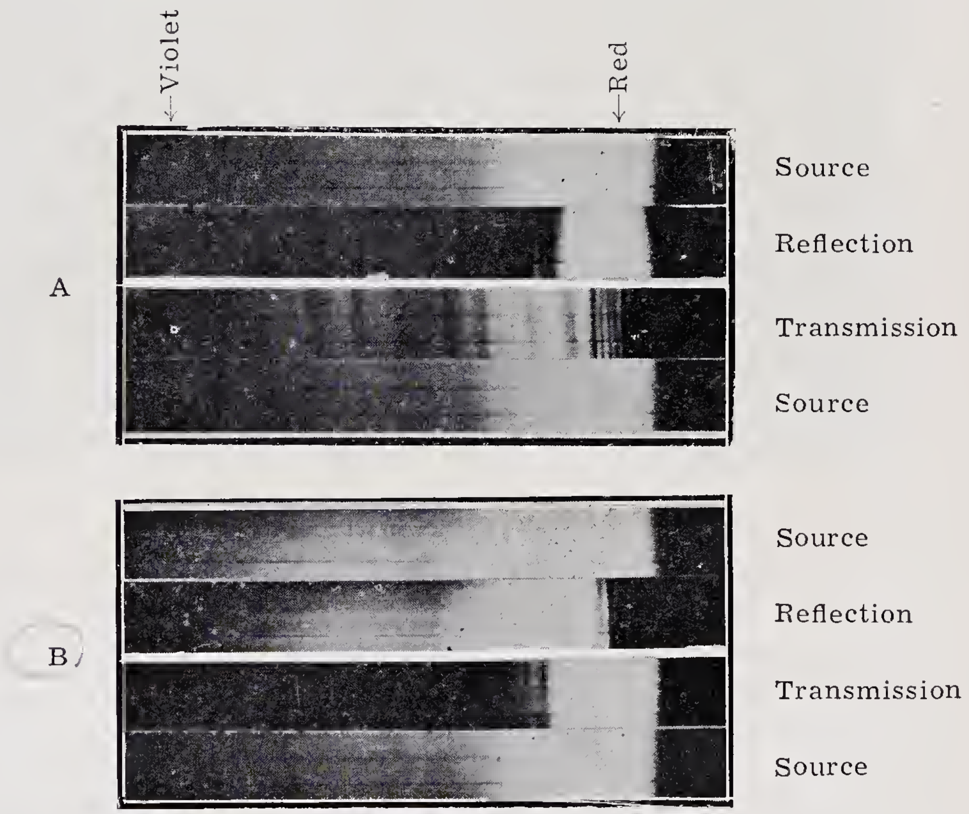

Fig. 29
Spectrograms of Ancient Decomposed Glass

analogous to those obtained with the stratified media already discussed. The spectra of the second kind are evidently of a different nature. They are analogous to those encountered when we examine the light transmitted by a thin film enclosed between highly reflecting surfaces, in which case most of the light is reflected except certain narrow regions in the spectrum which are transmitted. It would appear that in such cases the laminations have a specially large reflecting power.

Modern glassware when buried underground or exposed to chemical action often develops iridescence. But this is superficial and the transmission colours are poor. Examined by reflected light under the microscope, however, some beautiful effects may be observed. A distinctive type of decomposition often seen is one in which the surface of the glass is divided up like a map into areas of uniform colour with sharply

Fig. 30

Crater-like Forms in Decomposed Glass

defined boundaries. In other cases, again, we have numerous small cavities pitting the surface and appearing by reflected light as circular rings of colour. These may be separate but may and indeed often also appear closely grouped together over extensive areas. Glass which has been attacked severely frequently exhibits numerous hollow cup-like cavities adjoining each other, the walls of such cavities being in many cases themselves pitted by still smaller depressions. Some particularly remarkable cases may be seen in which the decomposition has proceeded symmetrically outwards from a nucleus on the surface and has at the same time spread downwards into the glass, forming a succession of thin concentric laminæ. The cavities formed by the removal of the decomposed material in such cases appear like circular craters which go down in a series of terraces to the deepest area at the centre, these terraces exhibiting brilliant colours by reflected light. Numerous such craters may be seen adjoining each other in Fig. 30 in which a few craters are also visible in which the material resulting from decomposition is still *in situ.**

The porous texture of glass which has been chemically rendered iridescent may be prettily shown by placing a small drop

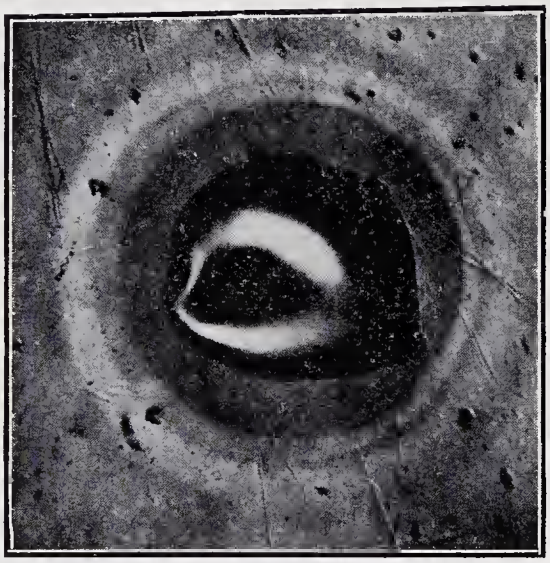

FIG. 31
Adsorption of Monobrom-naphthalene by Iridescent Surface

of monobrom-naphthalene on the surface and observing it by reflected light (Fig. 31). Surrounding the drop appears a circular area of film saturated with liquid, while beyond this is seen a succession of dark and bright rings which indicate a variation in the quantity and distribution of the liquid adsorbed by the film. These rings exhibit varied colours which are even more striking than the colour of the part of the film free from liquid. Monobrom-naphthalene being non-volatile, the pattern seen round the drop of it is static. The case is, however, different when a quickly-

* C. V. Raman and V. S. Rajagopalan, *Proc. Ind. Acad. Sci.*, 1939, 9, 371.

spreading and volatile liquid such as benzene is used; the colour patterns due to its adsorption and subsequent evaporation change with great rapidity.

The Lippmann Photographic Films.—Besides the four examples of a laminated structure giving iridescence which we have dealt with in detail, there are several others which would have been well worthy of discussion in these pages, had space permitted. For instance, the metallic colouration of many beetles has been shown to be due to the presence of stratified layers in their wing cases.* The brilliant colouration exhibited by many birds, insects and fishes may, at least in some instances, be of the same general nature. Amongst artificially produced periodic structures, pride of place must be given to the well-known Lippmann films in natural colour obtained by the action of stationary light waves on photographic films. The theory set out in the preceding pages gives a satisfactory explanation of the experimental facts observed with such films. The appearance for instance, of subsidiary bands *on one side only* of the principal maximum in the spectrum with films intended to exhibit a monochromatic reflection is clearly in accord with the theoretical expectations.‡ (See Fig. 12 above).

The frequency with which laminated structures are forthcoming in nature is rather remarkable. The mechanism giving rise to such structures appears to be of a varied nature, *viz.*, rhythmic crystallisation, multiple twinning, periodic precipitates, colloidal aggregation, and the natural processes of biological growth. Perfect regularity in the laminations is not always to be expected, nor is it essential for the production of vivid colour. Indeed, a lack of regularity may enable the medium to give a sensibly total reflection and a corresponding extinction over a wider range of wave-lengths than would otherwise be possible, and thereby to enhance the intensity of the colours observed without notably diluting their purity. This feature is of particular importance in those cases where the reflecting power of the individual laminations is small and a great many of them are present.

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* Rayleigh II, *Proc. Roy. Soc.*, A, 1923, 103, 233.
‡ R. V. Subramanian, *Proc. Ind. Acad. Sci.*, 1941, 13, 467.

LECTURE II

DIFFRACTION OF LIGHT

THE PRINCIPLES of geometrical optics indicate that the propagation of light is influenced by the presence of obstacles in its path in a manner determined by the form and properties of the obstacles. A polished metallic sphere, for instance, would reflect the light falling upon it and hence would cast a shadow. More complicated would be the effects due to a transparent obstacle, *e.g.*, a drop of water, as both the reflections and refractions at its surface would have to be considered. The determination of the resulting light intensity everywhere in the field in a case of this kind is a problem in the theory of diffraction. It is evident that the indications of geometrical optics must be supplemented by those of the wave-theory, in view of the possibility of interference arising between rays of light which cross each other after traversing different paths. In the case of the metallic sphere, for instance, the entire field outside the region of shadow should exhibit interferences between the incident and reflected rays. Similarly in the case of the liquid drop, the interferences between the direct and refracted rays would, in addition, need to be considered. The question also arises whether the conclusions derived from ray optics remain valid in all circumstances. In the particular case of the metallic sphere, for instance, the geometric shadow is sensibly perfect in the vicinity of the sphere, but at a sufficient distance behind it, observation reveals the presence of a bright spot of light at the centre of the shadow. How is this fact to be reconciled with the concepts of ray optics?

It is evident that the study of diffraction raises a fundamental issue, namely, the relationship between the ray and wave concepts of light. An adequate discussion of diffraction phenomena necessarily includes a consideration of these two aspects of optical theory and a reconciliation between them. The geometric approach provided by ray optics has the merit of simplicity and enables us to obtain an intuitive grasp of the phenomena. Hence, instead of considering diffraction phenomena as falling solely within the scope of wave-optics and as indicative of a failure of geometrical optics, it is a much more satisfactory procedure to bring diffraction within the scope of ray optics by appropriately framing our fundamental concepts. How this may be done is illustrated by the optical effects produced by a corrugated surface which we shall now consider in some detail.

Refraction of Light by a Corrugated Surface.—The phenomena which we shall now consider may be experimentally studied with a ripple tank, utilizing the fact that the surface of a liquid agitated by a linear system of ripples of definite frequency and wavelength is a corrugated surface. A narrow slit illuminated by a mercury lamp and followed by a collimating lens may be used as the source of light. The beam after passing through the liquid

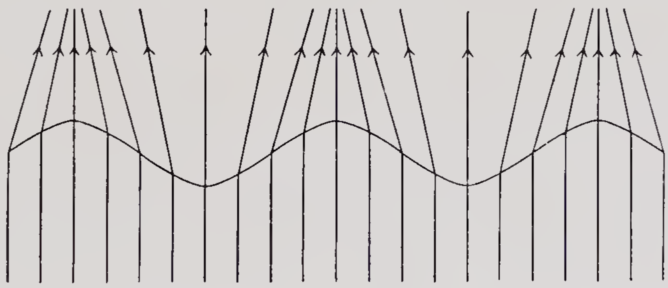


FIG. 32
Rays Emerging from Ripple Tank

surface or after reflection by it is viewed through a telescope focussed for parallel rays. We may consider here a parallel beam of light which passes vertically upwards through a ripple tank containing water. Fig. 32 represents the geometric courses of the rays of light on emergence from the liquid. The rays would be divergent over the concave areas and convergent over the convex areas of the surface, but at a sufficiently great distance would be divergent throughout. These elementary considerations of ray optics describe correctly the effects observed in the immediate vicinity of the surface. But at points sufficiently removed from the surface, they no longer represent the facts correctly.

We may ask ourselves, why is the geometric theory valid near the surface of the liquid and why does it apparently fail at points sufficiently far removed from it? The answer to these questions must be found in the fact that the disturbance on emergence from the liquid surface is no longer a simple train of plane waves. As remarked on page 5 in our discussion on the interference of light, *the principle of rectilinear propagation of light and the principle of interference do not in any way contradict each other.* But if apparent contradictions are to be avoided, it is necessary that we should recognise only waves of constant type on the one hand, and the rays normal to them on the other, as the proper basis for the description of the optical field. In our present problem, for instance, we should analyse the disturbance on emergence from the liquid surface into its component plane waves. The subsequent movement of such plane waves along their respective normals would furnish a description of the optical effects which is equally correct from the wave and the ray points of view and which is valid for every part of the field, both near and far from the surface of the liquid.

We have already seen (Fig. 2 on p. 4), that two plane wave-trains superposed on each other result in a stratification of the amplitude and phase of the disturbance in the field. The spacing 2D of such stratifications measured in a plane bisecting the angle between the wave-fronts is given by the formula $2\,D \sin \psi = \lambda$. This relation may be readily generalised. The superposition of sets of wave-trains travelling in various directions lying in a common plane and making angles ψ_n with some fixed direction in it, the relation $2\,D \sin \psi_n = n\lambda$ being satisfied, ($n = 0, \pm 1, \pm 2$, etc.), would result in the most general type of disturbance in which the amplitude and phase vary periodically in the plane normal to the fixed direction. Hence, putting $2D = \lambda^*$, where λ^* is the wave-length of the ripples, we may represent the light emerging from the liquid surface by such a set of plane waves travelling along the directions ψ_n given by the formula $\lambda^* \sin \psi_n = n\lambda$, ($n = 0, \pm 1, \pm 2$, etc.). Accordingly, when the light is viewed through a telescope focussed for infinity, a set of monochromatic images of the original light source would be observed on either side of the original direction of the light beam. The directions and intensities of these images would correspond to the plane waves into which the disturbance emerging from the liquid surface has been analysed. It will be noticed that the formula is the well-known one for the diffraction spectra due to a grating, but it has been obtained here directly from fundamental concepts, without introducing the Principle of Huyghens.

Fig. 2 on page 4 gives us other useful indications regarding the geometric character of the optical effects to be expected in our problem. Consider the pair of wave-trains proceeding along the directions $\psi_{\pm n}$. As shown by the figure, the space periods of the resultant disturbance perpendicular to the fixed direction and parallel to it are respectively $\lambda/\sin \psi_n$ and $\lambda/\cos \psi_n$. Hence, the resultants of the wave-trains ψ_0, $\psi_{\pm 1}$, $\psi_{\pm 2}$, $\psi_{\pm 3}$, etc., have space periods ∞, λ^*, $\lambda^*/2$, $\lambda^*/3$, etc., along any plane normal to the fixed direction, thus being in strict harmonic relationship. Along the fixed direction, however, the "wave-lengths" of the resultants are λ, $\lambda/\cos \psi_1$, $\lambda/\cos \psi_2$, $\lambda/\cos \psi_3$, etc. As these lengths are not identical the resultant disturbance would fluctuate as we proceed away from the liquid surface along the light beam. It can be readily shown that the disturbance would undergo a periodic cycle of changes, repeating itself completely when we advance a distance $2\lambda^{*2}/\lambda$ or any multiple of it, away from the surface. This is verified on putting $p\lambda = (p - n^2)\lambda/\cos \psi_n$, where p is an integer, and using the approximation

$$\cos \psi_n = (1 - \psi_n^2/2) = (1 - n^2\lambda^2/2\lambda^{*2}).$$

We obtain immediately $p\lambda = 2\lambda^{*2}/\lambda$. In other words, the result of superposing the waves $\psi_{\pm 1}$ on ψ_0 would repeat itself *once* when we advance a distance $2\lambda^{*2}/\lambda$, while that of superposing $\psi_{\pm n}$ on ψ_0 would repeat itself n^2 times within the same range. The appearance of the light field would, therefore, show fluctuations of a complex periodic character as we move away from the liquid surface, the spacing of the whole cycle being $2\,\lambda^{*2}/\lambda$.

An expression is readily found for the amplitudes of the plane wave-trains into which the disturbance emerging from the liquid surface is analysed. The light vector in the emergent wave when the surface of the liquid is plane may be taken proportional to

$$\sin\,(2\pi\,\nu t - 2\pi\,z/\lambda) = \sin\,Q.$$

The retardation of phase produced at a given epoch by ripples of amplitude 'a' progressing along the y-axis is

$$2\pi\,a\,(\mu - 1)/\lambda\cdot\cos\,2\pi\,y/\lambda^*.$$

We shall denote this for brevity by $v\cdot\cos\phi$. Accordingly, the expression for the light vector as modified by the presence of the ripples is proportional to $\sin\,(Q - v\cos\phi)$. This may be expanded and written in the form

$$\begin{aligned} J_0(v)\sin Q - J_1(v)\,&[\cos\,(Q+\phi) + \cos\,(Q-\phi)] \\ &- J_2(v)\,[\sin\,(Q+2\phi) + \sin\,(Q-2\phi)] \\ &+ J_3(v)\,[\cos\,(Q+3\phi) + \cos\,(Q-3\phi)] \\ &+ J_4(v)\,[\sin\,(Q+4\phi) + \sin\,(Q-4\phi)],\ \text{etc.} \end{aligned}$$

It is easily verified that the terms $\sin Q$, $\cos\,(Q\pm\phi)$, $\sin\,(Q\pm 2\phi)$, etc., represent plane waves travelling along the directions we have already indicated as ψ_0, $\psi_{\pm 1}$, $\psi_{\pm 2}$, etc. The intensities of these waves are accordingly proportional to $J_0^2(v)$, $J_1^2(v)$, $J_2^2(v)$, etc. As extensive tables of the Bessel functions are available, these quantities may be readily found for any assigned value of v. It may be remarked that, as is to be expected,

$$J_0^2(v) + 2\,J_1^2(v) + 2\,J_2^2(v) + 2\,J_3^2(v) + \ldots\ldots = 1,$$

so that the incident energy is merely redistributed amongst the different spectra.

Experimental Verification of Theory.—The behaviour of the Bessel function when the order and the argument are varied is well known.* The changes in the configuration of the diffraction

* See, for instance, Jahnke-Emde, *Tables of Functions*, Second Edition, 1933, Section XVIII.

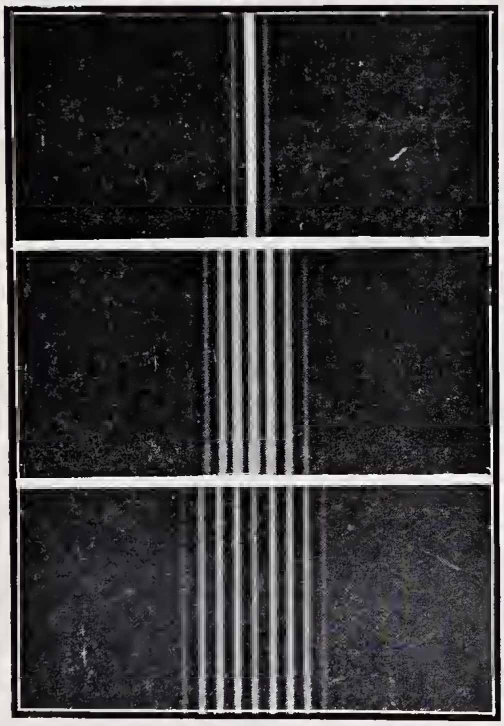
pattern with increasing values of v can, therefore, be readily visualised. When v is zero, we have only the central component. As v increases, the first orders begin to appear and increase in intensity while the intensity of the central component steadily falls off. The second order spectra then begin to appear. With further increase of v, a stage is reached when the first order spectra are very conspicuous, the second order spectra fairly strong and the third orders begin to appear, while the central component has nearly vanished. For still larger values of v, the second orders are stronger than the first, while the central component has reappeared and the third orders are in fair strength. Further changes of the same general nature occur for larger values of v, the spectra fluctuating in intensity, and the higher orders gaining at the expense of the lower ones.

FIG. 33
Relative Intensities of Spectra due to Corrugated Wave

The relative intensities of the spectra for various values of v from 0 to 8 are represented in Fig. 33 which is taken from a paper by Raman and Nath,* dealing with the somewhat analogous case of a beam of light which has passed through a liquid column carrying ultra-sonic waves. As will be seen from the figure, spectra of increasingly higher orders continue to appear as v becomes larger and take up the greater part of the energy, though

* C. V. Raman and N. S. Nagendra Nath, *Proc. Ind. Acad. Sci.*, 1935, 2, 406.

some of the spectra of lower orders still have fair intensities. The fluctuations in the intensity of any particular order with varying v, and of the spectra of different orders with constant v are both characteristic features of the case. The disappearance of particular spectra corresponds to the zeroes of the Bessel function of the orders concerned and furnish a sharp criterion for the value of the maximum phase-retardation which is operative. It is worthy of remark that the increase in the number of spectra and consequent enlargement of the angular width of the pattern with the increasing amplitude of the ripples, as shown by Fig. 33, roughly correspond with the increasing divergence of the rays on emergence from the liquid surface which is indicated in Fig. 32.



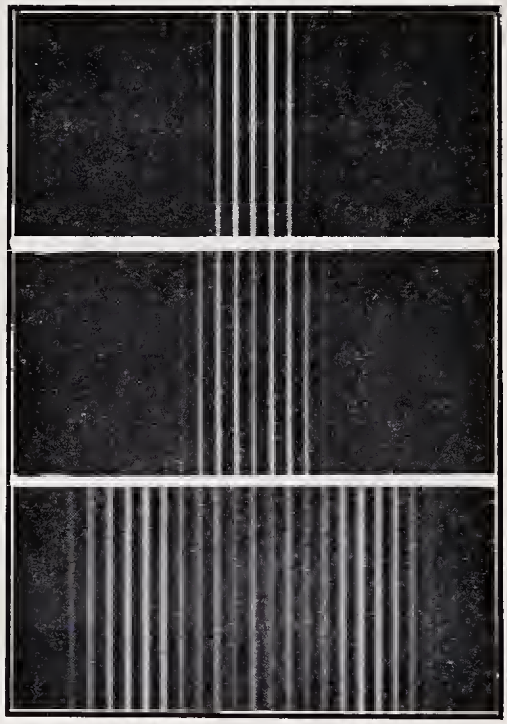


FIG. 34 FIG. 35

Diffraction of Light by Ripples on Water

Figs. 34 and 35 reproduce a series of photographs‡ obtained with a ripple tank and exhibit a beautiful concordance with the theoretical results. It should be remarked that these pictures were obtained with *progressive* ripples, the boundaries of the tank

‡ D. S. Subbaramiah, *Proc. Ind. Acad. Sci.*, 1937, 6, 333.

being too far away to give a disturbing reflection. Theory indicates that the diffraction pattern by *stationary* ripples would be of a different character. This is readily understood, because the relative intensities of the different orders of spectra depend on the maximum retardation of phase produced by the ripples. This is a constant quantity for a progressive motion, but varies periodically with time for a stationary oscillation of the liquid surface. In the latter case, the diffraction pattern as recorded on the photographic plate would be a time average in which the special features depending on the particular value of v would have been almost completely smoothed out. It is noteworthy that in either case, the diffraction patterns can be seen without stroboscopic aid. Indeed, stroboscopic illumination would make no difference in the diffraction pattern as observed with progressive ripples. With a stationary ripple pattern, however, the diffraction pattern seen would change with the phase at which the flashes of illumination are given. By altering this phase, it should be possible to follow the changes in the structure of the pattern corresponding to all the values of v from zero upwards to the maximum.

As the corrugation of the liquid surface produces changes of phase but no appreciable changes of amplitude in the light beam on its emergence, a microscope focussed on the surface would fail to give any indication of the existence of the ripples. When the plane of observation is moved away from the surface, however, alternations of light intensity would develop, which may be observed with stroboscopic aid for progressive ripples, and without such aid for a stationary ripple pattern. In the latter case, what we observe is an average intensity effect. The spacing of the pattern seen would therefore be $\lambda^*/2$, while for progressive ripples, the spacing would have the full value λ^*. As remarked earlier, the nature of the pattern would vary in a cyclic manner with the movement of the plane of observation. The spacing of the complete cycle is $2\lambda^{*2}/\lambda$ for a progressive wave, but if we ignore a lateral displacement of the pattern and consider only its general appearance, the period of the cycle would be one half of this, namely, λ^{*2}/λ. If, therefore, the plane of observation coincides with the liquid surface or is removed from it by a distance

λ^{*2}/λ or a multiple thereof, the ripples would be unobservable, while at intermediate positions, complex patterns would be seen the nature of which depends on the amplitude of the ripples. For stationary patterns seen without stroboscopic aid, the wave-length for the average intensity effect is effectively halved, so that the period of the cycle is only $\lambda^{*2}/2\lambda$. The general theory of visibility of periodic structures including the case of ultra-sonic waves and of optical gratings for which $\lambda^{*} \gg \lambda$ has been given by Nagendra Nath.§

Fresnel and Fraunhofer Patterns.—An application of the principle that the ray-optical and wave-optical descriptions of a light-field should be completely equivalent enables us to find the effect of restricting the aperture of a beam of light in any manner. To illustrate the essential features of the problem, we consider the comparatively simple case of the passage of light through a plane diffraction grating made up of parallel strips of equal width which are alternately transparent and opaque. Exactly as in the case of a corrugated refracting surface discussed earlier, we analyse the disturbance emerging from the grating into sets of plane waves travelling in various directions, starting from the disturbance at its surface as determined by its assumed properties.

The waves incident normally on the plane grating are represented by the expression A $\sin 2\pi(\nu t - z/\lambda)$. At the plane of the grating ($z=0$), this reduces to A $\sin 2\pi\nu t$, and we assume that this is also the disturbance emerging from the transparent strips, while over the opaque strips the disturbance vanishes. The light-field thus described is periodic over the surface of the grating with wave-length λ^{*} and may be therefore represented by its Fourier expansion

$$\frac{1}{2}\,A \sin 2\pi\,\nu t + \sum_{s=1,3,5,\ldots} 2\,A \sin 2\pi\,\nu t\;\frac{\sin 2s\,\pi y/\lambda^{*}}{s\pi}$$

We now reintroduce the co-ordinate z, and write the emerging disturbance in the form

$$\frac{1}{2}\,A \sin 2\pi\,(\nu t - z/\lambda) \pm \sum_{s=1,3,5,\ldots} \frac{A}{s\pi} \cdot \cos 2\pi\,(\nu t - z/\lambda \mp sy/\lambda^{*}).$$

§ N. S. Nagendra Nath, *Proc. Ind. Acad. Sci.*, 1936, **4**, 262.

The first term represents the undeviated plane waves of diminished strength emerging from the grating, while the others represent a series of diffraction spectra in which the even orders are missing. These spectra appear with equal amplitudes but with opposite phases in directions equally inclined to the primary beam on either side of it. The amplitudes of the diffracted plane waves are inversely proportional to the order of the spectrum, in other words, to the sine of the angle of diffraction. Considering the situation at the edges $b=0, \pm\lambda^*, \pm 2\lambda^*$, etc., on the surface of the screen, we notice that the diffracted plane waves traverse each of these edges in various directions but in identical phases. The diffracted disturbance may therefore be regarded as made up of sets of *cylindrical* waves diverging normally from these edges with an amplitude inversely proportional to the sine of the angle of diffraction. A similar situation also presents itself at the intermediate edges $y = \pm\lambda^*/2, \pm 3\lambda^*/2$, etc., except that the phases are now reversed. It follows that the cylindrical diffracted waves diverging from an edge have *opposite* phases according as they appear on the illuminated or the dark side of it. The diffraction spectra emerging from the grating may be considered as the result of the interferences of these cylindrical waves. On this view, the non-appearance of the spectra of even order is a consequence of the cylindrical waves from the equidistant edges being alternately in opposite phases.

The result which thus emerges, namely, that the boundary between light and shadow at the edge of a screen is a source of diffracted radiation having opposite phases on its two sides, evidently does not depend on the particular disposition of the edges in the case considered nor on their being infinitely extended or straight. It is indeed valid generally, for edges of finite length as also for curved edges. Further, the assumption that the alternate strips are completely opaque is also not essential, since a sudden transition of any kind—in amplitude or phase or both—on the two sides of a boundary of arbitrary form in a light-field would give rise to similar effects. The recognition of these consequences of optical theory is the key to an understanding of the diffraction phenomena which arise from the passage of light through apertures or its obscuration by obstacles of arbitrary form and nature. They enable us to describe these phenomena in geometric terms related to the form of the apertures or obstacles. Further, they lead us naturally to an understanding of the more recondite aspects of diffraction theory, including especially the influence of the material and thickness of the screens, and the configuration of the edges (*viz.*, whether they are sharp, wedge-shaped or rounded-off) on the observed phenomena.

A procedure closely analogous to that described above may be adopted to find the effect of passage of light through a single slit. We start by assuming a series of parallel slits of width a at regular intervals λ^* and then pass to the limit when $\lambda^* \rightarrow \infty$. As in the case considered above, a Fourier expansion gives the result of the passage of the light through such a grating in the form of a series. It is readily shown that this may be written in the form

$$\frac{2\,\mathrm{A}\,a}{\lambda^*} \cdot \left[\frac{1}{2} + \sum_{s=1,2,3,\ldots}^{\infty} (-1)^s \cdot \frac{\sin(\pi a \sin\theta_s/\lambda)}{(\pi a \sin\theta_s/\lambda)}\right]$$

where $\lambda^* \sin\theta_s = s\lambda$. As λ^* tends to ∞, the diffraction spectra appear in more closely contiguous directions, and their number being great, we may neglect the first term in comparison with the rest. On squaring, we obtain an expression of the familiar type for the intensity in the diffraction pattern given by a rectilinear slit. A similar expression is also obtained if we consider the diffraction pattern as the result of the interference of the cylindrical waves having opposite phases emitted by the two edges.

The validity of the treatment of diffraction problems outlined above is naturally subject to certain restrictions and in particular, depends greatly on the exactness with which the configuration and properties of any actual screen reproduce those assumed for the purpose of the Fourier analysis. It is evident also that the results of the analysis would progressively tend to deviate from the facts as the size of the structures considered approaches the wave-length of light. For, we would then actually have a terminated sequence of spectra and not an infinite sequence as assumed. These difficulties become much more acute when we consider cases in which the light is incident very obliquely on the screens or apertures.

Instead of analysing the disturbance emerging from an aperture into sets of infinitely extended plane waves, we may follow a different procedure and express it as the summation of sets of spherical waves having their origins continuously distributed over the area of the aperture. This, in fact, is the familiar approach to diffraction theory based on the Principle of Huygens due to Fresnel which is generally adopted in treatises on optics. In this treatment, the intensity at any point in the light-field is expressed as a surface integral taken over the illuminated area of the aperture and then evaluated. Apart from its historic interest and its usefulness in certain cases for purposes of computation, it is evident that this classic procedure has not much to recommend it from a physical point of view. It postulates secondary sources of radiation at points of space where there are no real sources and no material particles which can serve as secondary sources. Indeed, if we examine the matter closely, we find, on carrying out the summation of the effects of the postulated secondary sources, that they disappear from the picture, leaving only secondary radiations having their origins on the boundary of the aperture. That this is the case will be shown a little later when we consider the problem of diffraction by apertures or obstacles with curvilinear boundaries.

Diffraction by an Equilateral Aperture: The fundamental role played by the boundaries of an aperture in determining the physical and geometric characters of its diffraction pattern may be suitably illustrated by a simple case, namely, that of an aperture bounded by three sharp straight edges in the form of an equilateral triangle.

The light from a point source passing normally through such an aperture placed at a distance from it and after diverging further falls on a photographic plate; the pattern thus recorded with short exposures is reproduced as Fig. 36 and with large exposure as Fig. 37. Fig. 38 is a greatly enlarged reproduction of the pattern recorded when an image of the light source as seen through the aperture is brought to a focus on the photographic plate by a convex lens. On comparing Fig. 36 and Fig. 38, we notice that the former exhibits trigonal and the latter hexagonal

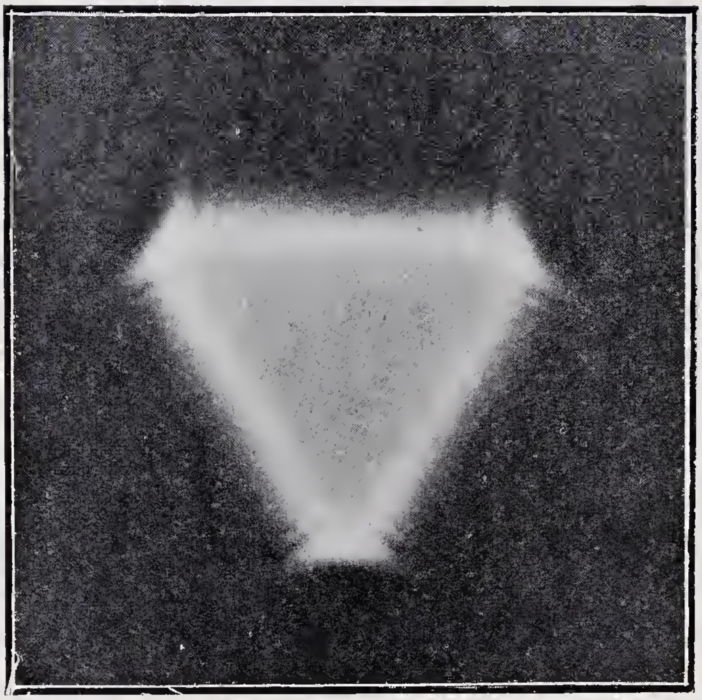


FIG. 36
Fresnel Pattern of Equilateral Aperture
(Weakly Exposed Photograph)

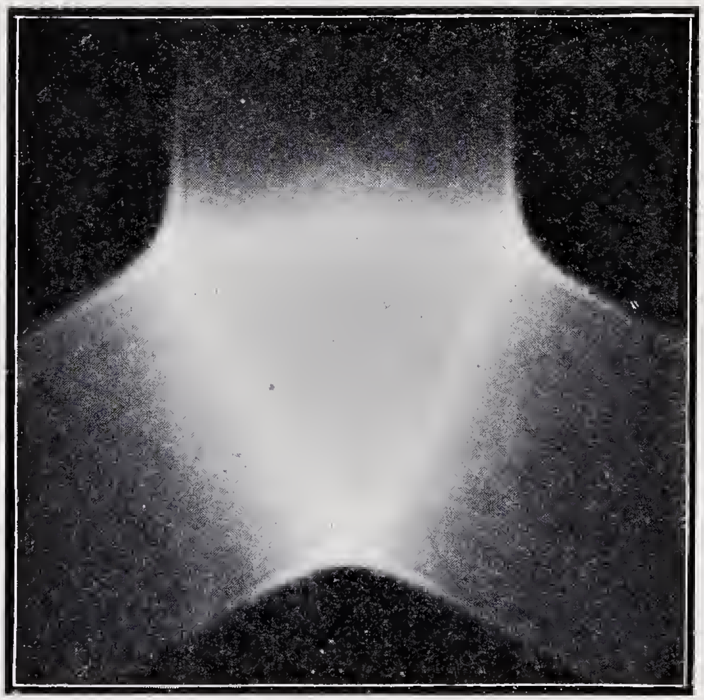


FIG. 37
Fresnel Pattern of Equilateral Aperture
(Strongly Exposed Photograph)

symmetry. The complete dissimilarity of the Fresnel and Fraunhofer patterns which these pictures suggest is negatived by a comparative study of Figs. 37 and 38. We then realise that the fainter outlying parts of the Fresnel pattern which are recorded on a strongly-exposed plate present marked similarities with the features observed in the Fraunhofer patterns.

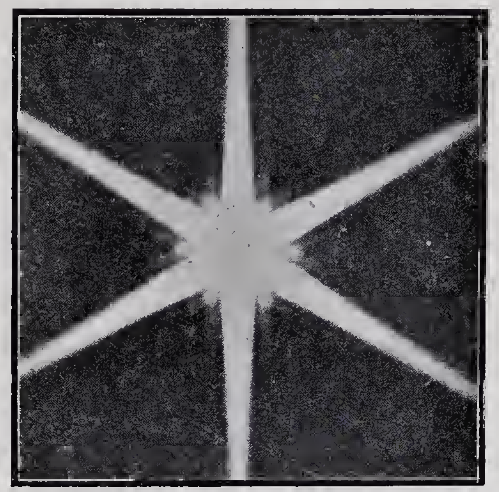


FIG. 38
Fraunhofer Pattern of Equilateral Aperture

The Fresnel pattern illustrated in Fig. 36 and Fig. 37 may be completely described by the statement that in the strongly illuminated triangular area, spherical waves of light passing directly through the aperture are superposed upon and interfere with the cylindrical waves diverging from each of its three edges, while in the fainter outlying regions, these cylindrical waves appear by themselves. The interference of the cylindrical waves from each edge with the primary spherical waves gives the bands of alternating intensity which are noticed running parallel to the same edge. Outside the central illuminated area, an area of hexagonal shape is noticed where the cylindrical waves from all the three edges are seen superimposed. Further out still, we have a star-shaped hexagonal pattern where they appear superimposed in pairs; still further out, they appear individually as streamers. Running parallel to the edges of these faintly illuminated areas, we observe bands of alternating intensity which are most prominent in the vicinity of the vertices of the triangle; in this neighbourhood, they take the form of hyperbolic curves having their asymptotes normal to the sides of the triangle.

The Fraunhofer pattern illustrated in Fig. 38 may be described as the result of the collapse and disappearance of the central illuminated area in Fig. 37, consequent on the lens bringing the spherical waves which pass through the aperture to a geometric focus. In other words, the Fraunhofer pattern is due exclusively to the radiations from the edges of the aperture. Where the radiations from all the three edges are effectively superposed, we have the central hexagonal area of the pattern. Surrounding this, features are visible which are due to the cylindrical waves being superposed on each other in pairs and giving observable interferences. Further out still, the effect of each edge is observed by itself in the arms of the six-rayed star which forms the most conspicuous feature of the pattern. The rays of this star arise from the cylindrical waves diverging on either side of each edge; owing to the astigmatism of these waves, the broad streaks to which they give rise in Fig. 37 have contracted laterally (but not longitudinally) into the narrow streaks seen in Fig. 38. The fainter bands running parallel to the rays of the star are essentially similar in their origin to the bands running normal to the edges seen in the Fresnel pattern. *They arise from the interference with each other of the cylindrical waves from different parts of the same edge.* Along the central line of each ray of the star, the whole of the corresponding edge is effectively in the same phase. But when we move away from the central line, the radiations from different parts of the edge no longer agree in phase. We may then replace the line source parallel to the edge by point sources of diffracted radiation, one at each end; these give the interferences running parallel to the rays of the star. The Fraunhofer pattern may indeed with justice be described as arising from the interferences of radiations from three point sources placed respectively at the three vertices of the triangle. The amplitudes and phases of these radiations depend in a characteristic manner upon the angle of diffraction, thereby influencing the general appearance of the pattern to a notable extent.

Diffraction Caustics and Foci.—We shall now proceed to consider the special phenomena exhibited by apertures and obstacles with *curvilinear* boundaries. It is useful in the first place to remark upon the relationship between the effects produced by an opaque obstacle and by an aperture in an opaque screen when they have the same form and situation relatively to the source of light. The illuminated areas being complementary, the sum of the effects in the two cases would everywhere be the same, being that due to the source itself. Accordingly, if we subtract from the undisturbed effect of the light-source, that actually observed at any point in the shadow in one case, the difference would be the effect observed in the illuminated area in the other, and *vice versa.* The form of the boundary being the common feature, this reciprocal relationship indicates that we are dealing in both cases with the same physical phenomenon, namely, a diffracted radiation having its origin at the boundary. That this appears as a positive contribution in one case and as a subtractive effect in the other, or *vice versa,* is readily understood, since the boundary radiation has opposite phases on the shadowed and illuminated sides respectively.

We have now to define more precisely the character of the radiation from a diffracting edge of arbitrary form. Here again, the essential features of the case may be deduced from the simplest assumptions. Spherical waves are assumed to diverge from a point-source Q, and at a distance D from it, meet an opaque screen covering the wave-front except over an aperture bounded by a curve of arbitrary form. We represent the disturbance emerging from the aperture as a summation of spherical waves having their origins distributed continuously over its area.* Accordingly, the disturbance reaching a point P on a distant screen is written as the integral

$$\iint \frac{A}{\lambda R} \sin 2\pi \left(\nu t - R/\lambda\right) dS$$

* The variation of the amplitude of the assumed spherical waves with their direction of propagation is unimportant for our present purpose and is, therefore, ignored.

where dS is an element of area on the aperture at the point O (say), and R is the length OP. It is readily shown that

$$dS = R\,dR\,d\epsilon \cdot D/PQ$$

where ϵ is the angle between the plane OPQ and some fixed reference plane passing through PQ. Integrating with reference to R, we obtain

$$\int_0^{2\pi} \frac{A \cdot D}{2\pi \cdot PQ} \cos 2\pi\,(\nu t - R/\lambda)\,d\epsilon.$$

R now signifies the distance from P of points on the boundary of the aperture and is, therefore, to be regarded as a function of ϵ. The integration with respect to ϵ from 0 to $2\,\pi$ may be written as an integration over a complete circuit of the boundary of the aperture; ds being an element of arc on the boundary, and ϕ the angle which it makes with the plane passing through PQ and the element, we obtain

$$\int \frac{A \sin \phi}{2\pi\, R \sin \theta} \cdot \cos 2\pi\,(\nu t - R/\lambda)\,ds,$$

where θ is the angle between the incident ray reaching the element ds and the diffracted ray starting from it and reaching the point of observation. Thus, it appears that *each line-element of the boundary is a source of diffracted radiation, its strength being inversely proportional to the sine of the angle of diffraction and directly proportional to the sine of its inclination to the plane of diffraction.*

The foregoing result is quite general and may be used to evaluate the intensity in both Fresnel and Fraunhofer patterns, it being always remembered that the integral represents the disturbance originating at the boundaries of the aperture or obstacle, and does not include the undisturbed effect of the light source. The latter, if present, must therefore be added to the expression. It will be noticed that the contribution from each element of the boundary becomes large and changes sign when θ passes through zero. It is this reversal of phase of the radiation from the edge that secures the observed continuity of the illumination when we pass from the region of shadow to the illuminated region in diffraction patterns of the Fresnel type. The *amplitudes* of the radiations received at any point in the field from different parts of the edges depend principally upon the angles θ and ϕ, while their *phases* vary with R. It follows that the resultant effect would be contributed mainly by the parts of the edge for which θ is small and R is a maximum or a minimum. The latter condition automatically ensures that ϕ is $\pi/2$ or $3\pi/2$ and $\sin\phi$ is therefore numerically a maximum. Ordinarily, therefore, the diffracted radiation has its origin principally at the point or points on the edge where this runs perpendicular to the rays reaching the points of observation. If, therefore, we draw a series of normals at various points on the geometric boundary between light and shadow on the receiving screen, these normals also define the directions in which the corresponding points on the edge are most effective in diffracting light.

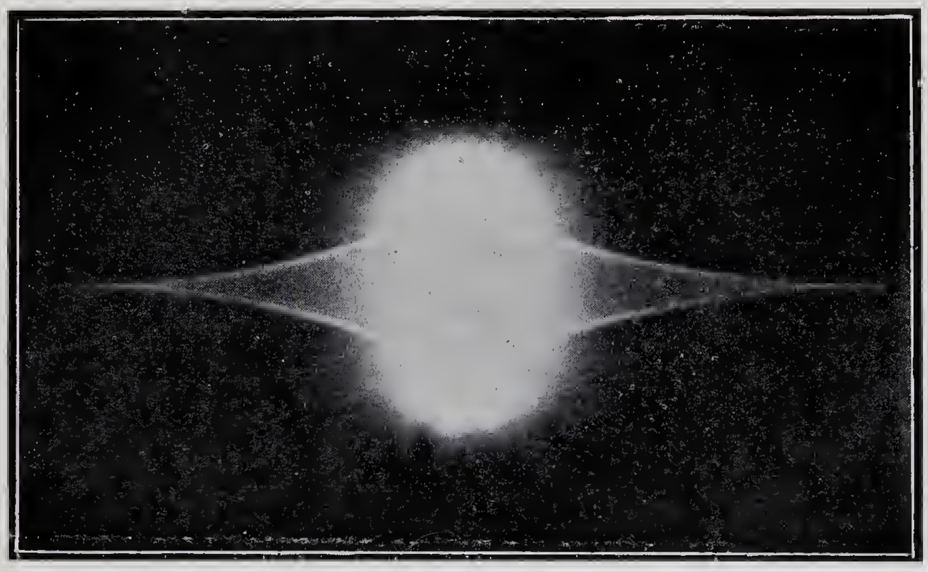


FIG. 39
Diffraction Caustic of Elliptic Aperture

In the language of geometrical optics, a focus is a point at which a set of reflected or refracted rays intersect, while a caustic is a curve to which a set of such rays is tangential. These concepts may evidently be applied *mutatis mutandis* to the rays diffracted by edges in the manner discussed above. Hence, provided the edges are sharp and smooth, besides being curved, we should be able to observe diffraction foci or diffraction caustics at the points where the normals to the geometric boundary of the shadow intersect or touch each other respectively.

Perhaps the best known example of a diffraction focus is the bright spot observed along the axis of the shadow of an opaque circular disc thrown by a point source of light. A smooth spherical obstacle also gives a similar bright spot, but with a notably different colour and intensity, for reasons which we shall consider more closely later.* It is worthy of remark that the edge of a circular *aperture* also gives a bright spot along the axis which may be rendered visible by blocking out the superposed illumination received directly through the aperture. It should also be mentioned that an *incomplete* circular edge also gives a bright spot, though naturally of inferior sharpness and intensity.

The considerations set out above indicate that the evolute of the shadow of a curved boundary is the geometric figure along which its diffraction caustic is formed. That this is actually the case was demonstrated by observations with an elliptic boundary made by the present writer many years ago.‡ A circular opacity polished surface held so that it reflected light obliquely proved an excellent substitute for an elliptic aperture. Less satisfactory was a circular disc or a circular aperture with a sharp edge cut in a thin metal sheet and held obliquely. Qualitatively, the geometric evolute was found to be the locus of maximum intensity of the diffracted light. More exactly, the evolute represented the geometric limit within which the radiations from the concave part of the curved edge were concentrated. The curve of maximum intensity lay along and inside the evolute, being accompanied by other and weaker interferences running parallel to it (see Fig. 39). These are due to the diffracted rays crossing each other at various small angles inside the evolute.

Fig. 40 shows the Fresnel diffraction pattern given by an elliptic aperture in monochromatic light. The intensity in the diffraction pattern outside the elliptic area is mostly concentrated within its geometric evolute as indicated by theory. The pattern is crossed by two sets of interferences, one of them running

* C. V. Raman and K. S. Krishnan, *Proc. Phys. Soc.*, 1926, 38, 350.
‡ C. V. Raman, *Phys. Rev.*, 1919, 13, 259.

parallel to the evolute and arising in the manner already explained, while the other set is transverse to the evolute and arises from the interference of the effects due to the convex and concave parts of the edge.

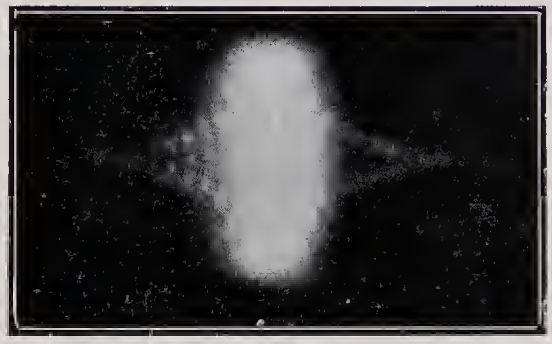


FIG. 40
Fresnel Pattern of Elliptic Aperture

Diffraction caustics are strikingly exhibited in the shadow of a disk with an undulating margin* (a nickel one-anna coin), two photographs of which taken with short and long exposures respectively are reproduced in Fig. 41. The diffraction pattern

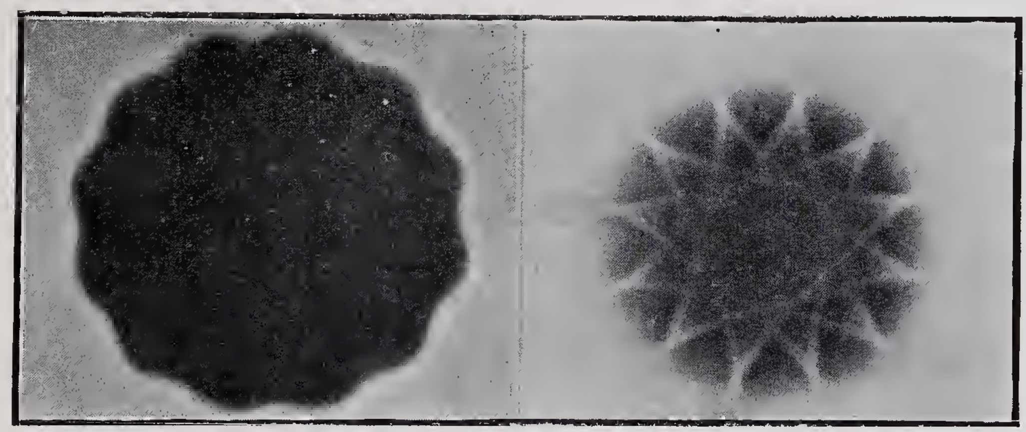


FIG. 41
Diffraction by an Undulating Edge

within the shadow which is recorded with the longer exposures follows the geometric form of the evolute of the undulating boundary. It exhibits a marked intensity at the cusps which may

* S. K. Mitra, *Phil. Mag.*, 1919, 38, 289. Other cases of interest, notably that of a disk with milled edges are illustrated in this paper.

be regarded as foci, as well as a concentration of the luminosity along the branches of the evolute, with subsidiary interferences parallel to them. The effects in the vicinity of the cusps are very similar to those noticed when two branches of a caustic formed by refraction meet, *e.g.*, in the case of an obliquely held transparent cylinder.*

The Heliometer Diffraction Figures.—The case of a semi-circular aperture has a special interest in its application to the form of the star-images seen in the heliometer; this instrument is a telescope with a divided objective, the two halves of which are capable of displacement relatively to each other along their common diameter. The case also offers an excellent illustration of the power of the geometrical method in discussing the configuration of Fresnel and Fraunhofer diffraction patterns and the relations between them. We shall here first consider the Fraunhofer pattern, and show how its geometric features may be deduced from the semi-circular form of the boundary.

We denote the angle between the incident and diffracted rays by θ, and the complement of the angle which the plane containing these rays makes with the diameter of the semi-circle by ψ. When $\psi = 0$, the plane of diffraction is normal to the diameter; the radiations from the elements of the straight edge have then the maximum amplitude, while their phases are in agreement for all values of θ. Accordingly, these radiations give a long bright streak in the pattern along a line perpendicular to the diameter. When ψ is not zero, the phases disagree, and the interferences then arising would result in alternate minima and maxima of intensity running parallel to the streak along $\psi = 0$. As in the case of the equilateral aperture considered earlier, we may regard these interferences as due to point-sources of equal strength and opposite phases placed at the extremities of the edge. When $\psi = \pi/2$, the radiations from the straight edge vanish completely.

Considering now the curved part of the boundary, the amplitude of the radiations from a line element is a maximum at the point where the edge runs perpendicular to the plane of diffraction, and diminishes gradually to zero at the points where it

* T. K. Chinmayanandam, *Phys. Rev.*, 1918, 12, 314.

runs parallel to that plane. The phases of the radiations from the line elements as received in the focal plane would, on the other hand, be stationary at the former point and alter at an increasing rate as we approach the two latter points. The

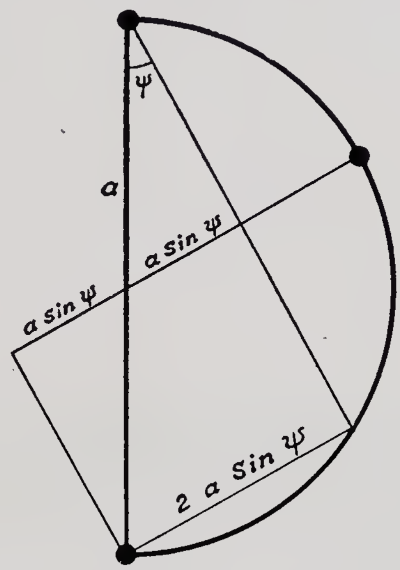


Fig. 42
Diffraction by Semi-Circular Boundary

resultant effect of all the line-elements may, therefore, be represented by that of a source placed at the point of stationary phase, supplemented by two equal sources located at the ends of the semi-circle. These latter vanish when $\psi = 0$, but become increasingly important as ψ alters in either direction; in the limiting cases when $\psi = \pm \pi/2$, they represent the entire effect of the boundary.

Thus, in general, the radiations from the straight and curved parts of the edge, taken together, may be replaced by the effect of three point-sources placed at the positions indicated in Fig. 42, namely, one on the curved edge and two at the corners of the aperture. The optical paths traversed by the radiations from these sources in reaching the focal plane would differ from each other by the three quantities $(a + a \sin \psi) \sin \theta$, $(a - a \sin \psi) \sin \theta$ and $2a \sin \psi \sin \theta$, as will be seen from Fig. 42. If we denote $a \sin \theta$ by ζ, the quantities ζ and ψ may be used to define the polar co-ordinates of a point in the focal plane. The interferences of the three sources, considered in pairs, would lie along the lines whose equations are $2\,\zeta \sin \psi =$ constant, $(\zeta + \zeta \sin \psi) =$ constant, and $(\zeta - \zeta \sin \psi) =$ constant. The first equation represents the long streaks perpendicular to the diameter already mentioned, while the two latter represent sets of parabolæ whose axes are *parallel* to the diameter of the semi-circle, but whose curvatures are oppositely directed with reference to the set of straight bands represented by the first equation. These features are beautifully exhibited by the photograph of the Fraunhofer pattern reproduced in Fig. 43. The three sets of interferences and their intersections completely determine the form of the pattern and the distribution of intensity in it.

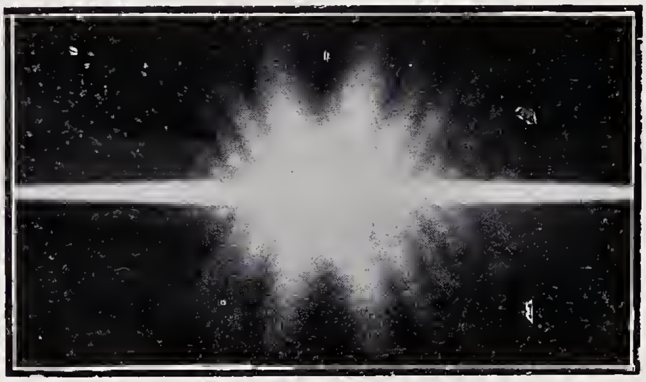


Fig. 43
Fraunhofer Pattern of Semi-Circular Aperture

As already remarked, the effect of the straight edge vanishes when $\psi = \pi/2$, while that of the semi-circle is equivalent to two sources placed at the ends of the diameter. The situation is then the same as for a complete circular boundary, except that we have only one-half its effect. In other words, in the plane of the diameter, the pattern is identical with that of a complete circular aperture with the intensities reduced to one-fourth their values. When $\psi = 0$, the effect of the curved edge reduces to that of a single source placed at its midpoint; the interferences of this with the effect of the diameter would result in the straight bands due to the latter fluctuating in intensity along their length. These fluctuations diminish both relatively and absolutely, and ultimately disappear, as we move out along these bands. For, the elements of the straight edge remain in phase with each other, while those of the curved edge fall out of phase in an increasing measure as θ is increased. The effect of the latter, therefore, falls off much more rapidly than that of the former.

The line integrals over the straight and curved parts of the boundary are readily evaluated and by their summation, the intensity variations in the pattern may be found quantitatively. The former integral is

$$\int_{-a}^{a} A/2\pi f\cdot \sin\theta \times \cos(Z - 2\pi s\cdot\sin\psi\cdot\sin\theta\cdot/\lambda)\cos\psi\, ds$$

$$= Aa/\pi f\cdot\cos Z\cdot\cos\psi/\sin\theta\cdot\sin\xi/\xi$$

where f is the focal length of the lens, ξ stands for $2\pi a \sin\psi$. $\sin\theta/\lambda$ and Z for $2\pi(\nu t - f/\lambda)$. The latter integral is

$$\int_{-\psi}^{\pi-\psi} A/2\pi f \sin\theta\times\cos(Z - 2\pi a\sin\theta\cdot\sin\phi/\lambda)\sin\phi\cdot a\, d\phi.$$

When ψ is equal to $\pm\pi/2$, this reduces to

$$A\cdot\frac{\pi a^2}{f\lambda}\cdot\frac{J_1(2\pi a\sin\theta/\lambda)}{(2\pi a\sin\theta/\lambda)}\cdot\sin Z,$$

which is half the value for a complete circle. The effect of the semi-circular arc is evaluated very simply for the case when $\psi = 0$. For, the point-source to which it is then equivalent must be such that together with an equal source at the opposite end of a diameter, it should give the effect of a complete circle. Using the well-known semi-convergent expansion for the Bessel function $J_1(x)$, the amplitudes and phases of the sources may be so fixed that their interferences give everywhere the required intensity.*

Comparing the photographs reproduced as Figs. 43 and 44 respectively, we notice that the Fraunhofer pattern has both a horizontal and a vertical axis of symmetry, while the Fresnel pattern has only the former. This is an example of the general

* S. K. Mitra, *Proc. Ind. Assoc. Cult. Sci.*, 1920, 6, 1.

theorem that a Fraunhofer pattern always exhibits centro-symmetry, while the Fresnel pattern has no higher symmetry than the aperture itself. The centro-symmetry of Fraunhofer patterns

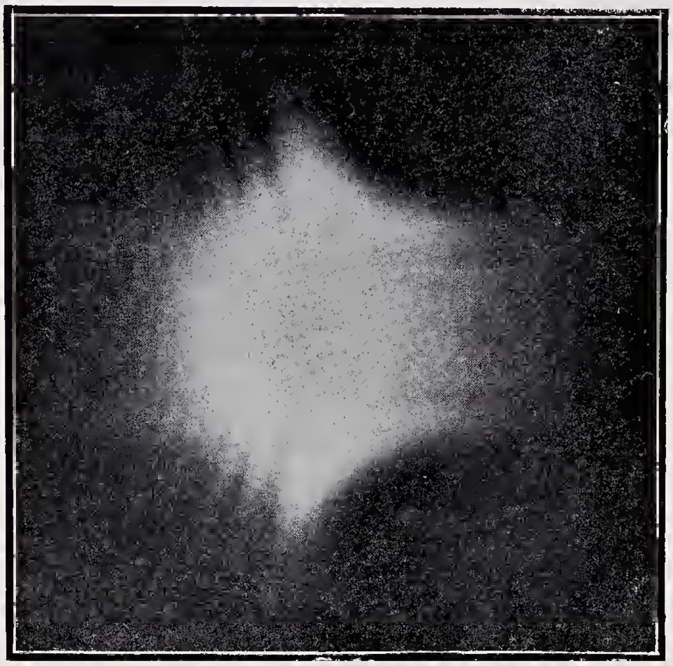


FIG. 44
Fresnel Pattern of Semi-Circular Aperture

is explained by the circumstance that they are due exclusively to the boundary radiations, the angle of diffraction vanishing at the centre of the pattern for all the elements of the edge; this is not the case for Fresnel patterns.

Various features are noticeable in Fig. 44 which may be explained by considering the radiations from the diffracting boundary and their mutual interferences. Those from the straight edge appear on both sides of the pattern as horizontal bands. Those from the curved edge are divergent on the left, but on the right converge to a focus and then diverge again. If longer exposures had been given, streamers would have been recorded diverging from the focus towards the right in various directions. The lack of symmetry of the Fresnel pattern about a vertical axis would then have been somewhat less conspicuous. The transverse bands seen in the fainter regions on both sides of the pattern arise from the interferences of the effects of the straight and curved parts of the boundary.

Diffracting Apertures in the Foucalt Test.—The general view of diffraction phenomena outlined in the preceding pages is strikingly confirmed and illustrated by the effects noticed when an illuminated aperture is observed by the aid of the light diffracted by itself. To enable this, an achromatic lens of good quality is employed to form a focussed image of a point source of light. An aperture of the chosen form is placed immediately after the lens and restricts the area of the emerging beam. After reaching a focus, the light enters a second achromatic lens which forms an image of the illuminated aperture on a distant screen. The character of this image is found to be greatly influenced by restricting the aperture of the second lens in any manner, the size and position of the openings in the focal plane through which the light enters the second lens determining the nature of the effects observed. The phenomena noticed are readily interpreted on the view that the light forming the Fraunhofer pattern in the focal plane of the first lens has its origin on the edges of the diffracting aperture. By varying the disposition of the openings which admit the light into the second lens, effects may be exhibited which demonstrate the various characters of the edge radiation to which reference has already been made, *viz.*, the sudden reversal of its phase in passing from interior to exterior diffraction, the dependence of its intensity on the angle of diffraction and the concentration of the intensity at special points on the edge, namely, the poles of the point of observation and sharp terminations on the edge.

When the opening of the second lens is not restricted in any manner, it gives an image of the illuminated aperture reproducing its form with full definition. A remarkable transformation of the image occurs when a small obstacle is placed at the region of maximum intensity in the Fraunhofer pattern, so as to block it out. The image then formed is a delineation of the boundaries of the aperture, the area within the boundaries appearing more or less perfectly dark. It should be remarked, however, that the edges of the aperture do not appear as bright lines in the image so formed. On the contrary, they appear as *perfectly dark lines* bordered by alternately bright and dark fringes on either side, provided the aperture of the second lens is symmetrically disposed with reference to the centre of the pattern. The reason for this fact is readily understood. As was remarked earlier, the Fraunhofer pattern is centro-symmetric by virtue of the interior and exterior diffraction by every element of the edge being of equal intensity but of opposite phases. Hence, when the light admitted into the second lens includes both interior and exterior diffraction to equal extents, the two radiations cancel each other by

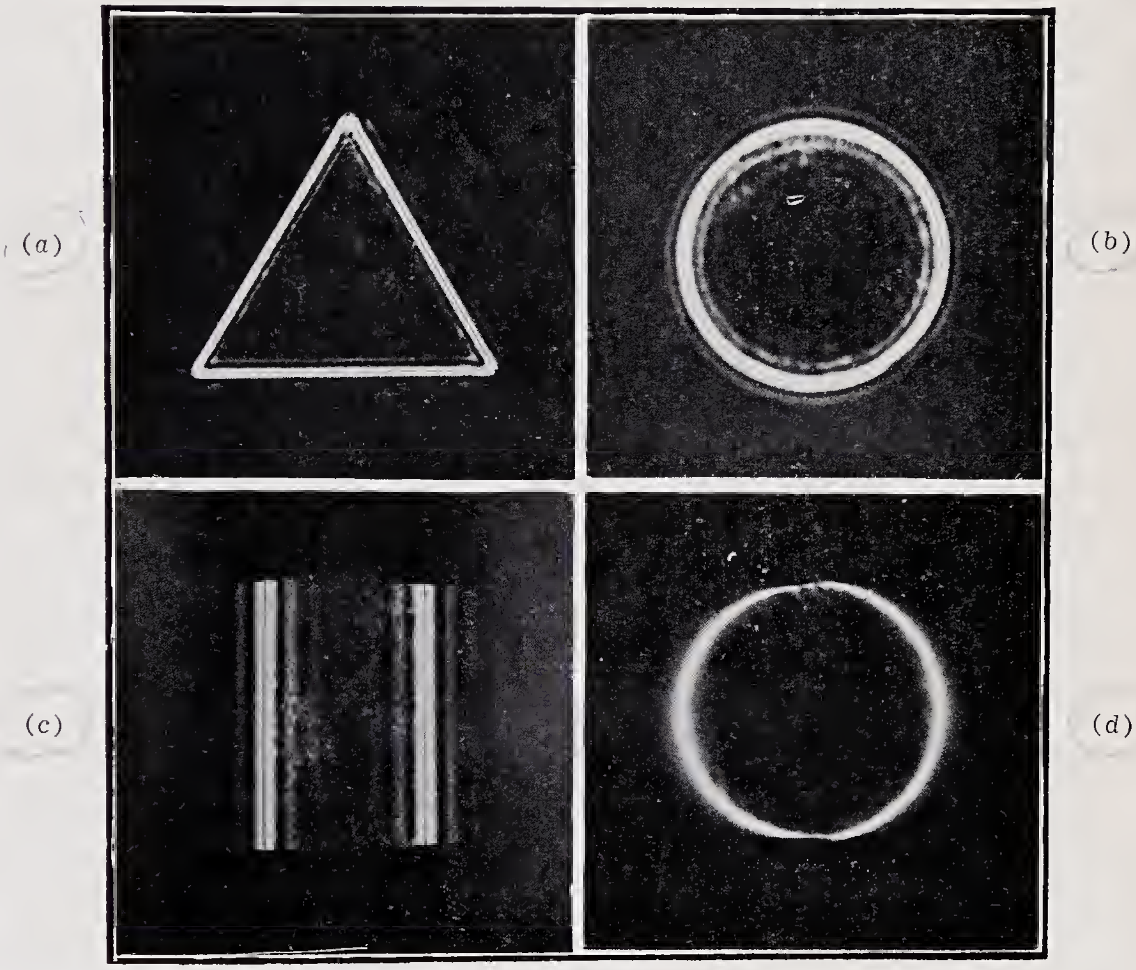


Fig. 45
Diffracting Apertures in the Foucalt Test

interference and give an image of zero intensity; the diffracted light appears in the interference fringes bordering the image of the edge on either side. If, on the contrary, the aperture of the second lens is not centro-symmetric but admits the light only on one side of the focus, this would no longer be the case. The image of the edge then appears as *a bright line* bordered by alternate dark and bright fringes.

When the second lens is completely covered except for a small opening which admits the light reaching some chosen point in the pattern, the luminosity of the edge is, in general, observed only at the poles of this chosen point. Taking, for instance, the case of a circular aperture, only two bright spots are seen, one at each end of a diameter. The intensity of these spots falls off with the increasing angle of diffraction as the opening is moved away from the focus. It is desirable that, in such observations, the opening used is not either too small or too large; in the former case the image becomes weak and diffuse, and in the latter case, the radiations from too great a length of the edge are admitted. Even when a small opening is used, the spots at the two ends of the diameter of a circular aperture lengthen into bright arcs and finally appear as semi-circles when the opening is brought close to the focal point. In this limiting position, the edge appears brightest where it is perpendicular to the plane of diffraction and is of zero intensity where it is parallel to this plane, as required by theory. Observations of this kind serve to illustrate the geometric theory of diffraction discussed in the preceding pages. An aperture of polygonal form, for instance, exhibits a luminous point at each of its corners which brightens and merges with the luminosity of the edge observed when the opening in the focal plane lies on the corresponding streamer in the pattern. A semi-circular aperture exhibits, except in special cases, three bright spots on its boundary in the positions indicated by Fig. 42, the relative brightness of these spots varying greatly with the position of the opening. In the particular case when the chosen position is on the horizontal axis of symmetry (Fig. 43), the entire diameter appears luminous, while if it is on the vertical axis, only two bright spots are seen at the ends of the diameter, as in the case of a complete circular aperture.

Figs. 45 (*a*) and 45 (*b*) are photographs of apertures as seen by diffracted light, the central rays being blocked out at the focus by a small sphere of wax; the edges appear in each case as dark lines bordered by bright lines. Figs. 45 (*c*) and 46 represent the appearance of the respective apertures when the direct rays are blocked out by a fine wire stretched across the Fraunhofer pattern through the focus. The broad fringes seen on

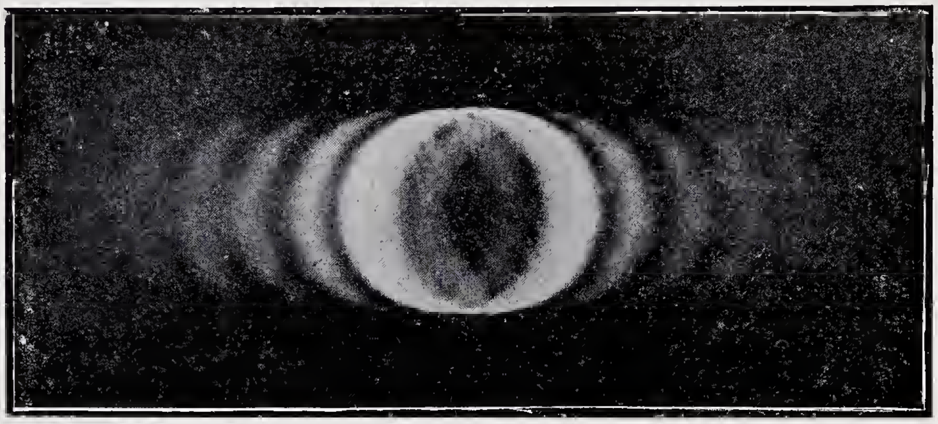


Fig. 46
Circular Aperture in the Foucalt Test

either side of the edge in these two figures are in striking contrast with the extreme sharpness with which the geometric form of the edge itself is recorded as a dark line. These features become intelligible when it is recalled that the breadth of the diffraction fringes depends on the diameter of the wire stretched in the focal plane, while the whole effective aperture of the second lens determines the sharpness with which the edge is depicted. Fig. 45 (*d*) represents a circular aperture as seen in the usual form of the Foucalt test, a knife-edge cutting off the light at the focus. It will be noticed that the edge now appars as a bright curve which is intense on the horizontal diameter and fades off towards the vertical.

The theory of the effects observed in the Foucalt test is readily worked out in the relatively simple two-dimensional case* of a rectangular aperture. The light vector at any point in the focal plane of the first lens is, in this case, proportional to

$$2 \sin Z \cdot \sin (2\pi/\lambda \cdot \alpha \cdot x)/x$$

where α is the semi-angular aperture of the opening which limits the area of the beam, and x is the co-ordinate of any chosen point

* Rayleigh I, *Scientific Papers*, 6, 455.

in the focal plane. We consider this disturbance as effective over the aperture of the second lens, and to find the effect due to it at the focal plane of the latter, we integrate it over the area, paying due regard to the phase differences which arise between the effects due to the elements of this area when the observation is in a direction making an angle β with the axis of the original light beam. The integral to be evaluated is

$$2\int dx\cdot\sin(Z+2\pi/\lambda\cdot\beta\cdot x)\ \sin(2\pi/\lambda\cdot\alpha\cdot x)\cdot/x$$

which may be written as

$$\sin Z\left\{\int dx\cdot\sin 2\pi/\lambda\cdot(\alpha+\beta)\ x/x+\int dx\cdot\sin 2\pi/\lambda\cdot(\alpha-\beta)\ x/x\right\}$$

$$+\cos Z\left\{\int dx\cdot\cos 2\pi/\lambda\cdot(\alpha-\beta)\ x/x-\int dx\cdot\cos 2\pi/\lambda\cdot(\alpha+\beta)\ x/x\right\}$$

The integrals have to be taken between the limits x_1 and x_2 of the aperture covering the second lens. They reduce to the well-known Si and Ci functions of which complete tables are available.* By assuming particular values for x_1 and x_2, the distribution of intensity in the focal plane of the second lens may be numerically worked out. In the case of the simple knife-edge test, for example, x_2 would be taken large and x_1 small, both having the same sign. The calculations show‡ that when the edge has covered only half the central band, the luminosity at the edges of the aperture is already six times greater, and when it has covered the whole of the central band, about 80 times greater than the illumination at the centre of the aperture. The computations* also show that the luminosity of the areas between the edges alternately increases and diminishes, the successive maxima becoming smaller, as the knife-edge advances. The fluctuations of colour which are observed simultaneously over the whole area of the aperture are thus satisfactorily explained. When the knife-edge traverses an ultra-focal plane of the first lens, the two edges of the aperture appear of different intensity—as is to be expected,

* Jahnke-Emde, *Tables of Functions*, 1933, p. 83.
‡ Rayleigh I, *loc. cit.*
§ S. K. Banerji, *Astrophysical Journal*, 1918, **48**, 50.

since we are then dealing with the Fresnel pattern of the aperture.* The great brightness and sharpness of the luminosity at the edges appear in these calculations as mathematical consequences of the behaviour of the Ci function which falls very steeply to an infinite negative value when its argument tends to zero. The intensity thus becomes very great in the directions $\alpha = \pm\beta$, provided x_1 is small and x_2 is large, both being of the same sign. On the other hand, when the apertures are symmetrically disposed, the Ci's being even functions of the argument cancel each other in the expressions for the intensity, while the Si's which are odd functions vanish for a zero value of the argument and reach a finite limiting value for large arguments. That the edges then appear as dark lines in the directions $\alpha = \pm\beta$ is thus readily explained, as also the sharpness of these lines when the full aperture of the second lens is operative. The case of a circular aperture can very similarly be dealt with in terms of the Si and Ci functions when the aperture of the second lens has a symmetric opening.‡

Diffraction by Sharp Metallic Edges.—The edge of a razor-blade held in a beam of light is observed to diffract light through large angles, appearing as a luminous line, when viewed either from within the region of shadow or from the region of light. The light thus diffracted is found to be strongly polarised, but in perpendicular planes in the two regions. Gouy, who was the first to notice these effects, experimented with edges of various metals and discovered that both the colour of the light diffracted into the shadow and its state of polarisation depend in a remarkable manner on the material of the edge and on the extent to which it has been rounded off in the process of polishing. When viewed through a double-image prism from within the shadow, only that image of the edge appears coloured which is more intense and is polarised with the magnetic vector parallel to the edge. The second image which is fainter and is polarised with the electric vector parallel to the edge, appears perfectly white. When the incident light is polarised in any arbitrary azimuth,

* S. K. Banerji, *Astrophysical Journal,* 1918, 48, 50.
‡ S. K. Banerji, *Phil. Mag.,* 1919, 37, 112.

the diffracted light is, in general, elliptically polarised. The explanation of these phenomena will now be discussed.*

Earlier in this lecture, it was shown from theoretical arguments that the edge of an opaque screen functions as a source of cylindrical waves; these diverge both into the region of shadow and into the region of light, but in opposite phases in the two regions, their amplitude being inversely proportional to the sine of the angle of diffraction. In establishing this result, the edge was regarded as a line of discontinuity between the area in which the full effect of the incident primary waves is present and the area in which it is completely cut off. An aperture with sharp edges in a very thin sheet of metal makes a fair approach to the situation here contemplated. It should be remarked, however, that such a sheet would also reflect backwards such of the light falling on it as is not absorbed. The boundary of such an aperture is, therefore, a discontinuity in two distinct ways, firstly in respect of the light passing through it, and secondly in respect of the light reflected by the surrounding area. Diffraction effects have, therefore, to be considered arising out of these two distinct

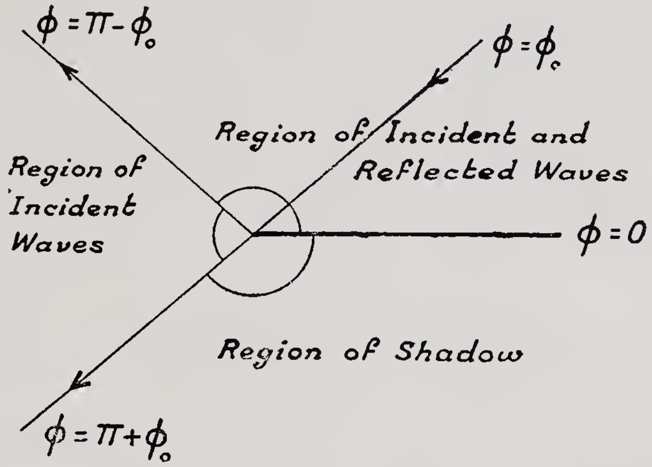


FIG. 47
Reflection by Half-Plane

processes. The reflection of light was ignored in our earlier discussions, but if it is taken into account, the colour and polarisation effects discovered by Gouy find a natural explanation.

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* C. V. Raman and K. S. Krishnan, *Proc. Roy. Soc.*, A, 1927, 116, 254.

We shall, in the first instance, consider the case of a plane metallic screen bounded by a straight edge. Plane waves of light travelling towards the edge and incident on it in the direction $\phi = \phi_0$ (see Fig. 47) may be represented by the real part of the expression

$$e^{ik\rho \cos (\phi - \phi_0) + ikct}$$

The waves reflected by the upper surface of the screen and receding from it are given by the real part of

$$-(C_s + iD_s)\, e^{ik\rho \cos (\phi + \phi_0) + ikct}$$

or of

$$+(C_p + iD_p)\, e^{ik\rho \cos (\phi + \phi_0) + ikct}$$

The complex amplitudes for the reflected wave are those characteristic of the metal for the particular angle of incidence. The alternatives refer to the cases in which the electric and magnetic vectors respectively are parallel to the edge of the screen for both the incident and the reflected waves.

We have now to find expressions for a set of cylindrical waves radiated from the edge of the screen, the superposition of which on the incident and reflected wave-trains would give in the vicinity of the planes $\phi = \pi + \phi_0$ and $\phi = \pi - \phi_0$, effects which are the same as those indicated by elementary theory, but which would be valid also for large angles of diffraction. These requirements are met if we multiply the foregoing expressions by the factor S given by the equation

$$S = \frac{e^{i\pi/4}}{\sqrt{\pi}} \int_{-\infty}^{\sigma} e^{-i\sigma^2}\, d\sigma,$$

where

$$\sigma = \sqrt{2k\rho}\, \cos \tfrac{1}{2} (\phi - \phi_0), \text{ for the incident waves}$$

and

$$\sigma = \sqrt{2k\rho}\, \cos \tfrac{1}{2} (\phi + \phi_0), \text{ for the reflected waves.}$$

It is easily verified that the resulting products are solutions of the wave-equation in cylindrical co-ordinates. S tends to different values, namely, unity and zero, when σ is positive and negative respectively and is sufficiently large. The result of the multiplication is, therefore, to restrict the appearance of the incident and reflected wave-trains to the particular regions indicated in Fig. 47. The multiplier S is, however, not *exactly* unity or zero as the case may be; its *difference* from these values is given with all needful accuracy by the following expression (obtained by integration in parts):

$$-\frac{e^{i\pi/4}}{\sqrt{\pi}} \cdot \frac{e^{-i\sigma^2}}{2i\sigma}.$$

Substituting the values of σ in this expression and multiplying it with the expressions for the incident and reflected wave-trains, we obtain finally, by addition, the result

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{\rho}} \cdot e^{-i\,(k\rho - kct + \pi/4)} \cdot \left[\frac{1}{\cos\frac{1}{2}(\phi - \phi_0)} - \frac{C_s + \iota D_s}{\cos\frac{1}{2}(\phi + \phi_0)}\right]$$

or

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{\rho}} \cdot e^{-i\,(k\rho - kct + \pi/4)} \cdot \left[\frac{1}{\cos\frac{1}{2}(\phi - \phi_0)} + \frac{C_p + \iota D_p}{\cos\frac{1}{2}(\phi + \phi_0)}\right].$$

These expressions represent cylindrical waves diverging from the edge and having an amplitude varying with the direction of the observation, the angle of incidence of the light and its state of polarisation, and also dependent on the optical constants of the metal. The latter appear in the complex reflection amplitudes which are given by the formulæ

$$-(C_s + \iota D_s) = \frac{a - \cos\psi}{a + \cos\psi}$$

$$+(C_p + \iota D_p) = \frac{\omega^2 \cos\psi - a}{\omega^2 \cos\psi + a},$$

where ψ is $(\pi/2 - \phi_0)$, $\omega = n\ (1 - \iota k)$ and $a^2 = (\omega^2 - \sin^2\phi)$; n and k are the index of refraction and the absorption coefficient, respectively, of the metal. It will be seen that the cylindrical waves have large amplitudes near the boundaries of the geometric shadow and of the geometric reflection, and that the phases of the components which become large are reversed in crossing these boundaries. Further, the components bear the same relation of amplitude and phase to their respective parent waves. Their interference with these waves would, therefore, correctly describe the diffraction fringes observed in the vicinity of these boundaries. The expressions being also valid for the large angles of diffraction, they contain within themselves the explanation of the experimental facts concerning the intensity, colour and state of polarisation of the light diffracted by sharp edges.

The foregoing treatment of the problem of diffraction by a straight edge is based upon Sommerfeld's solution for the physically unrealisable case of a perfectly reflecting half-plane, but differs from it in taking the actual properties of the screen into account. It might be remarked that a metallic knife-edge is more appropriately regarded as a wedge than as a half-plane. The theoretical expressions (due to Poincare and others) for a perfectly reflecting wedge may, however, be modified in the same way by considering the actual reflecting power of the material of which it is made. Taking the wedge to be bounded by the surfaces $\phi=0$ and $\phi=\chi$, the edge effect is given by formulæ similar to those for a half-plane, $\frac{1}{4\pi}$ being replaced by $\sin\frac{\pi^2}{\chi}\Big/2\chi$, and the expressions within the square brackets by

$$\left[\frac{1}{\cos\frac{\pi(\phi-\phi_0)}{\chi}-\cos\frac{\pi^2}{\chi}}-\frac{C_s+\iota D_s}{\cos\frac{\pi(\phi+\phi_0)}{\chi}-\cos\frac{\pi^2}{\chi}}\right]$$

or by

$$\left[\frac{1}{\cos\frac{\pi(\phi-\phi_0)}{\chi}-\cos\frac{\pi^2}{\chi}}+\frac{C_p+\iota D_p}{\cos\frac{\pi(\phi+\phi_0)}{\chi}-\cos\frac{\pi^2}{\chi}}\right]$$

as the case may be. Writing $\chi=2\pi$, we immediately regain the preceding expressions for a half-plane.

The new feature introduced by considering the optical properties of the screen is that the edge radiation appears as a summation of two distinct effects. For normal incidence ($\psi=0$), the components due to reflection are numerically identical but are different in sign for the two possible states of polarisation. If, therefore, the incident light be unpolarised, the light diffracted through large angles would be partially polarised, and this effect would be most marked in directions nearly parallel to the surfaces of the screen or wedge, since the quantities added or subtracted would then be nearly equal numerically. The electric vector parallel to the edge would be the stronger component in the region of light or exterior diffraction, and would be the weaker component in the region of shadow or interior diffraction.

The incident light being white, the colour of the radiation diffracted by the edge would be determined by two considerations. The wave-length appears as a factor in the expression for the intensity, and thus would tend to make the diffracted light reddish, irrespective of the state of polarisation of the incident radiation. The colour would also be influenced, and in quite a different way, by the appearance of the optical properties of the metal explicitly in the expressions. Since the amplitudes of reflection occur with opposite sign in the two cases, the wave-lengths strengthened in one case will be weakened in the other, and *vice versa*. Thus, in the region of shadow, the colours for which the reflecting power of the metal is greater would appear strengthened in the electric vector perpendicular to the edge and weakened in the electric vector parallel to the edge. This effect would increase with the angle of diffraction, so much so that the favoured colours would become more and more marked in one component, and less and less marked in the other component, the further we proceed into the region of shadow. Similar effects should occur in exterior diffraction, but with the parallel and perpendicular components exchanging places in respect of both colour and polarisation.

If the incident light be plane-polarised in an azimuth inclined to the edge, the diffracted light would exhibit a rotation of the plane of polarisation as well as ellipticity, the latter appearing as a consequence of the change of phase in metallic reflection. From what has already been remarked about the effects observed when the incident light is unpolarised, it follows that both the rotation and ellipticity would depend on the nature of the screen, and in the case of strongly coloured metals would be notably a function of the wave-length of the light. The determination of these two quantities gives us both the ratio of intensities of the parallel and perpendicular components and the difference of phase between them, and would, therefore, enable a more stringent test of the theory to be made than would be possible when incident unpolarised light is employed.

The results of the foregoing theory are in general agreement with the original observations of Gouy. Quite recently, the subject has been very carefully investigated by M. Jean Savornin,* in whose memoir will be found also a complete bibliography of the subject and a detailed discussion of the earlier results of other workers. Savornin has quantitatively studied the phenomena and compared them with the results expected theoretically on the assumption that the screens are perfectly conducting half-planes or wedges, as also according to the modified formulæ in which the optical properties of the material are taken into account. The evidence is decisively in favour of the latter procedure. Indeed, Savornin's data for the variation with the wave-length of the rotation of the plane of polarisation and the ellipticity of the light diffracted by a razor edge, and by the same when covered with gold by cathodic deposition, show such a striking resemblance with the curves deduced from the theory and the known optical properties of steel and gold respectively, as to leave no room for doubt of the essential correctness of our formulæ. The ellipticity for a steel edge is found to be small and to diminish towards the red end of the spectrum, while the rotation increases at the same time, though only slowly. In the case of gold, the ellipticity shows a pronounced maximum at about 5,200 A.U., dropping off to smaller values at both smaller and longer wave-lengths, while the rotation exhibits a minimum at about 4,900 A.U., and rises very steeply towards the red end of the spectrum. These observations are in full accord with the indications of our theory.

The formulæ are equally successful in other respects. They completely explain the striking difference in colour of the two components of the diffracted light. They account for the rapid increase in the rotation of the plane of polarisation and in the ellipticity produced by tilting the plane of the diffracting edge

* J. Savornin, *Annales de Physique*, 1939, 11, 129.

away from the symmetric position in one direction or the other, the deviation of the diffracted rays remaining constant. They also explain in a quantitative manner the variation of these quantities and of the intensity of the diffracted light with the angle of diffraction. In making the comparison between theory and experiment, it is necessary to consider not only the optical constants of the material, but also the angle of the wedge, since the latter is found to influence the results very markedly, especially when this angle is at all considerable. It is also necessary to remember that the theory is strictly applicable only in the case of a perfectly sharp and straight edge—a state of affairs to which the observations show that a fresh razor blade may be a remarkably good approximation. It may be mentioned that the optical phenomena exhibited by such an edge are so striking and so easily observed that they should be personally familiar to every student of optics.

Diffraction by Semi-Transparent Laminæ.—As is well known, metals are ordinarily opaque to light, but in extremely thin layers transmit light. By cathodic deposition, or preferably by evaporation in vacuum, it is possible to obtain metal films of controlled thickness and consequently of any desired degree of transparency. By protecting part of the surface of a plate of glass or quartz by a razor edge held obliquely in contact with it, it is possible to coat the plate with a metallic film partially transmitting light and terminating in a straight edge*, leaving the rest of the plate clear. The same technique is also applicable for obtaining non-metallic films by evaporation in vacuum. The diffraction patterns of the Fresnel class due to semi-transparent laminæ bounded by a straight edge exhibit interesting features not observed either with opaque screens or with transparent laminæ. In the familiar diffraction pattern due to an opaque screen with a straight edge, we have a few fringes of rapidly diminishing visibility running parallel to the edge in the region of light, and a continuous illumination falling off steadily to zero intensity in the region of shadow. A semi-transparent film, on

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* N. Ananthanarayanan, *Proc. Ind. Acad. Sci.*, 1939, 10, 477.

the other hand, exhibits a great number of interference fringes in the region of shadow, besides the fringes of moderate visibility in the strongly illuminated region. As the thickness of the film is increased and its transparency thereby diminished, the first few fringes in the region of shadow become less clear, while the subsequent ones gain visibility in spite of the diminished intensity of light in the field. The fringes having the maximum visibility move further into the region of shadow with increasing thickness of the film, until ultimately they disappear altogether. Another interesting feature is that the appearance of these patterns alters greatly with the colour of the light. This is a consequence of the marked variation of the transparency of the films with the wave-length.

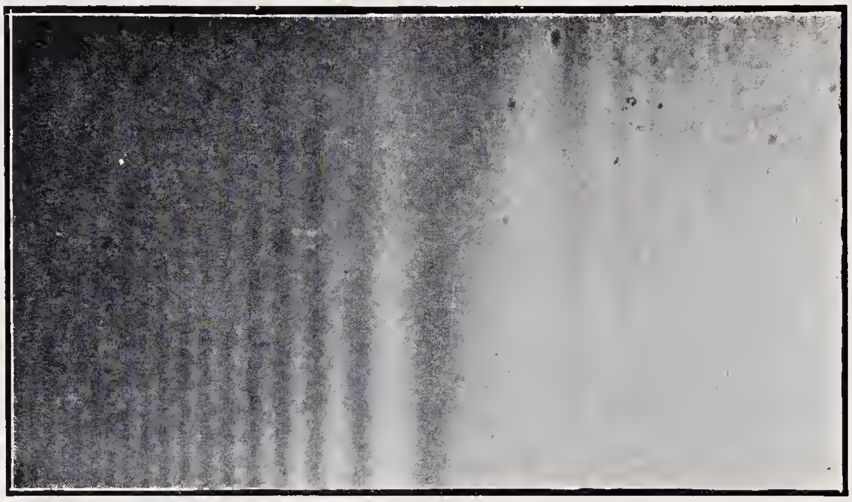


Fig. 48
Diffraction by a Thin Metallic Half-Plane

Fig. 48 reproduces the Fresnel fringes due to the edge of a semi-transparent silver film. To enable the effects observed on both sides of the edge to be seen simultaneously, a photometric wedge was set across the pattern when it was photographed. The fringes of low visibility on the strongly illuminated side, and the numerous fringes of high visibility in the shadow were thus recorded respectively in the upper and lower parts of the photograph. To exhibit the changes in the pattern with wave-length, the fringes may be set transversely on the slit of a spectrograph.

The dispersion by the instrument then indicates how the position and intensity of the fringes alter with wave-length. Silver has a region of comparative transparency in the near ultra-violet at about 3,250 A.U. The measurements by Minor show that the optical constants n and k change rapidly in the vicinity of this band, n falling steeply and k rising equally rapidly with increasing wave-length. The influence of these changes on the configuration of the diffraction fringes is shown by the spectrogram reproduced in Fig. 49. A marked displacement of the fringes on the two sides of the band at 3,250 A.U. and their practical disappearance inside it are the notable features revealed by this record.*

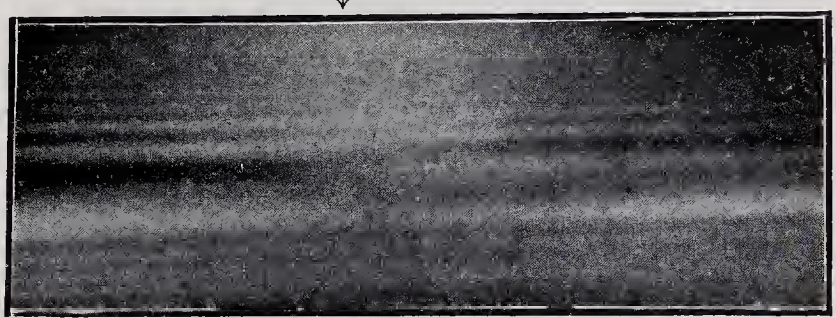
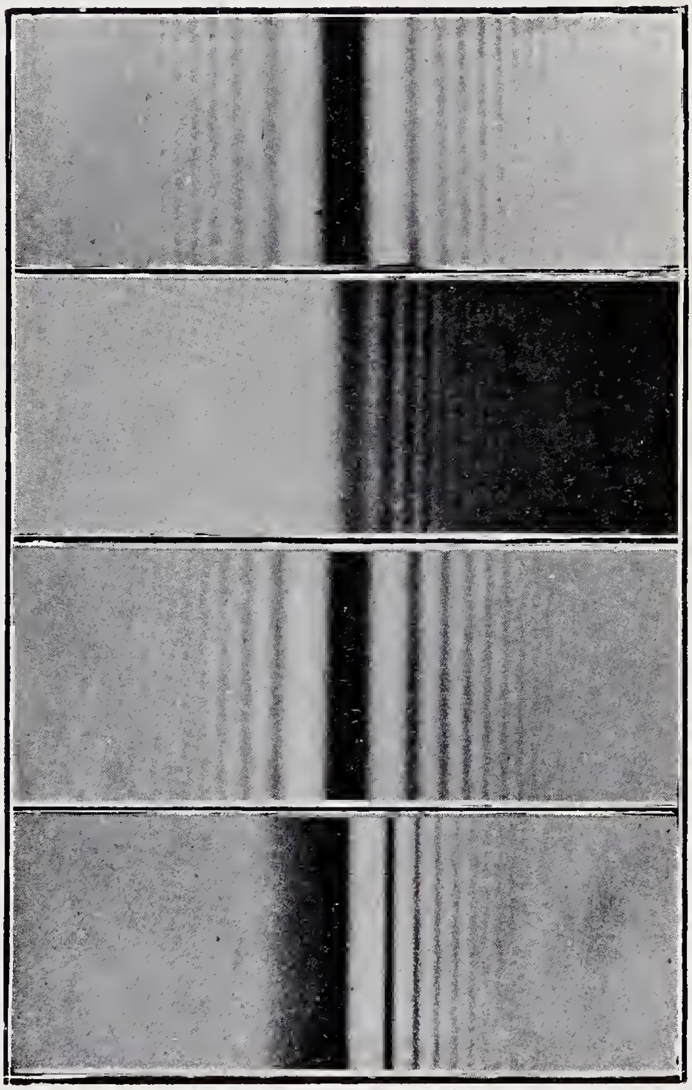


FIG. 49
Diffraction by Silver in the vicinity of 3250 A.U.

The phenomena described above become intelligible when we regard the Fresnel pattern as arising from the interference of the cylindrical waves having their origin at the edge of the film with the plane waves regularly transmitted on either side of it. The high visibility of the interferences observed in the region of shadow is a consequence of the light transmitted by the film and that diffracted by the edge being of comparable intensity. The thicker the film, the further we have to move into the region of shadow for this situation to occur, while both nearer and still farther, the interfering waves differ greatly in amplitude, thereby diminishing the visibility of the resulting fringes. The positions occupied by the maxima and minima of intensity depend on the geometrical differences of path between the interfering plane and

* N. Ananthanarayanan, *Proc. Ind. Acad. Sci.*, 1942, 14, 85. The photographs reproduced in Fig. 50 are also from his work.

cylindrical waves, due correction being made for their initial phase-difference, and especially for the change in phase of the light transmitted through the film. These considerations also serve to explain the remarkable variations in the positions and visibility of the fringes noticed in Fig. 49 as we pass along the spectrum, and especially in the vicinity of the band of transparency at 3250 A.U.


(a)

(b)

(c)

(d)

FIG. 50
Fresnel Patterns of Semi-Transparent Laminæ

By exposing a silver film having a sharp edge to the action of iodine*, it may be completely converted into a film of silver

* A tiny crystal of iodine placed on a silver film gives beautiful rings of colour.

iodide which transmits the red end of the spectrum freely, but is nearly opaque at the violet end. Fig. 50 (*a*) reproduces the Fresnel pattern given by such a film in red light. Its approximately symmetrical character indicates that it arises, at least in part, from a difference in phase between the waves transmitted on either side of the edge. Fig. 50 (*b*) which reproduces the Fresnel pattern observed in violet light is a striking contrast. Here, the part covered by the film appears as in deep shadow; the photographic exposure necessary to record the fringes seen resulted in those of low visibility appearing in the strongly illuminated part of the field being effaced by over-exposure. Figure 50 (*c*) is the Fresnel pattern of a somewhat thicker and therefore less transparent film in red light; this shows the fringes more clearly on one side than on the other. Fig. 50 (*d*) illustrates the importance of a sharp edge by exhibiting the effect of its absence on the Fresnel pattern due to a film of canada balsam; it will be noticed that the fringes have completely disappeared from one side and show rapid changes in visibility on the other side. This is the result of the film sloping off at its termination instead of having a sharply-defined edge.

Colours of the Striæ in Mica.—As is well known, mica has a perfect cleavage, and the sheets obtained by splitting the mineral exhibit a remarkable uniformity of thickness, as is shown, for instance, by the perfection with which they exhibit Haidinger's rings, and by the uniform colour and brightness of large areas of the surface of the sheet as seen by the reflected light of a mercury lamp. Whenever a variation of brightness or colour appears, it is sudden, indicating a sharply defined boundary at which a change of thickness occurs. These changes of thickness become vividly apparent when the mica is examined by the method of the knife-edge or Foucalt test. They then appear as *bright* lines, often exhibiting brilliant colours.* Observed in the same test but with a symmetrical aperture, the striæ are seen as *dark* lines bordered by bright coloured fringes on either side, indicating a reversal of phase of the edge waves analogous to that we have already

* C. V. Raman and P. N. Ghosh, *Nature*, 1918, 102, 205; see also P. N. Ghosh, *Proc. Roy. Soc.*, 1919, 96, 257.

noticed in the case of other diffracting boundaries (see Fig. 51). Microscopic examination shows that the laminar boundaries may be single or multiple *, and the phenomena which the striæ exhibit are found to depend on their precise nature in this respect. Single striæ are extremely sharp laminar edges and are, therefore, well suited for a critical study of the diffraction effects due to such edges. They diffract light through large angles §, exhibiting colour and polarisation effects which, in some respects, are analogous to and in other respects differ from those observed with sharp metallic edges. Examined by direct light in critical focus under high powers of the microscope, the edges appear as dark lines bordered by bright fringes,‡ but as we shall see presently, the observed facts are not reconcilable with elementary notions of diffraction theory. These circumstances lend interest to a detailed study of the phenomena and justify an attempt to explain them on the basis of more exact theory.$

When a sheet of mica is held in the path of a beam of light and the field of view behind it is examined through a lens, the striæ render themselves evident by the diffraction fringes of the Fresnel class to which they give rise. From the character of these fringes, it is evident that they arise principally from the difference in phase of the waves transmitted on either side of the boundary, though the difference in amplitude consequent on the difference in thickness also requires consideration. The central fringe in the Fresnel pattern is often brightly coloured. If, however, the difference in thickness be considerable, no colour is noticed, but the central fringe then appears darker than the rest of the field. When the slit of a pocket spectroscope is set transversely across the pattern, alternate dark and bright bands may be seen running obliquely through the spectrum at the centre of the fringe system. The bright bands determine the colour of the central fringe and if their number is not too great, it may be noticed that they correspond to the wave-lengths at which the outer diffraction fringes are very weak. These oblique

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* P. N. Ghosh, *Proc. Ind. Assoc. Cult. Sci.*, 1920, 6, 51.
§ I. R. Rao, *Ind. Jour. Phys.*, 1928, 2, 365.
‡ N. K. Sur, *Proc. Ind. Assoc. Cult., Sci.*, 1922, 7, 125.
$ C. V. Raman and I. R. Rao, *Proc. Phys. Soc. Lond.*, 1927, 39, 453.

bands in the spectrum indicate that the position of the central dark fringe shifts laterally with the change of wave-length. Using white light and a stria giving a sufficiently large path difference, this asymmetry averages out and should cease to be evident in the absence of spectroscopic aid. In practice, however, the fringes are often more prominent on one side of the pattern than on the other. There is little doubt that this is due to a disturbing factor, namely, the finite width of the striæ. Instead of a single sharp edge, we have a series of them like an echelon. In such a case, the diffraction fringes are stronger on the retarded side of the wave-front than on the other, a circumstance which is readily understood if we consider the form of the wave-front after its passage through the echelon.

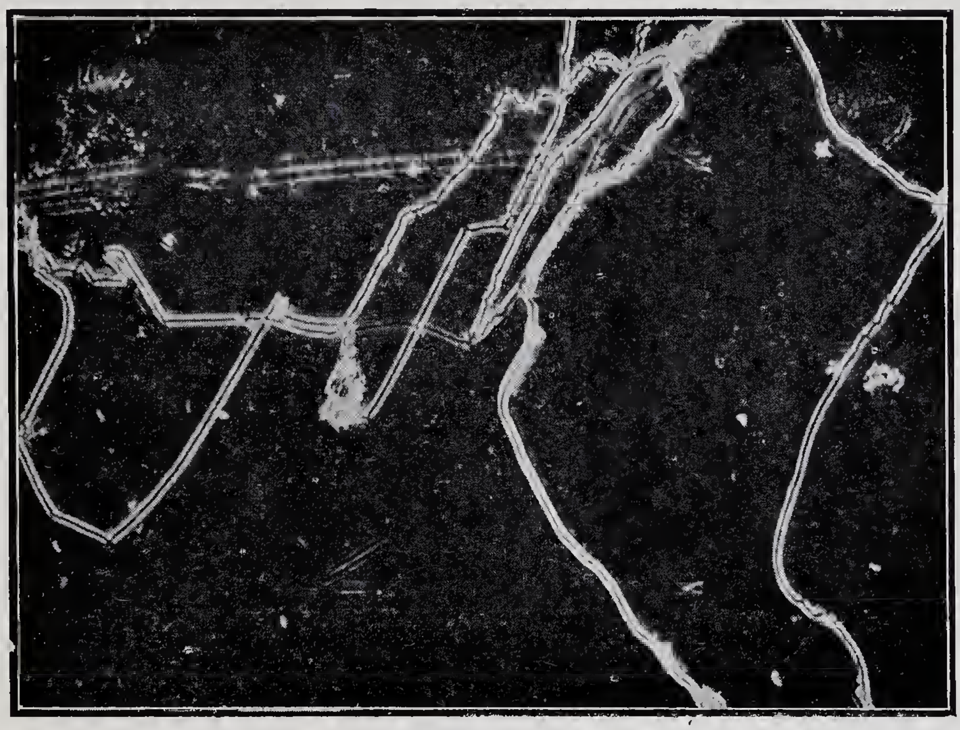

FIG. 51
Mica Striæ in the Foucalt Test

The Foucalt test is the most suitable way of examining the light diffracted by the striæ at comparatively small angles with the primary beam. The colours exhibited are complementary to those observed in the central fringe of the Fresnel patterns, and the changes produced by tilting the plane of the mica with reference to the incident light are also of a complementary character in the two cases. Thick striæ appear white in the Foucalt test. Spectral examination, however, with discloses an alternation of intensity with wave-length. This is readily explained, as the difference in phase of the waves transmitted on either side of the boundary would have a maximum effect when it represents an odd number of half wave-lengths and would have no effect when it represents an integral number of wave-lengths; only the small differences in amplitude would then be left to give any observable diffraction effect, according to the elementary theory.

For examining the light diffracted at larger angles, it is convenient, as in the case of metallic edges, to illuminate the stria and view it through a low-power microscope focussed on it. It then appears as a bright line exhibiting colour, but the latter is found to alter with the angle of diffraction in a manner which depends on the character of the stria, *viz.*, the phase-retardation on the two sides of the boundary and its micro-structure. The most striking and interesting results are those shown by single striæ. Seen in a direction nearly coinciding with the incident light, the colour is the same as when it is observed in the Foucalt test, *viz.*, complementary to the colour of the central fringe in the Fresnel pattern. But on tilting the microscope away from this direction, the colour alters in a continuous sequence which is not symmetrical with respect to the direction of the incident light. The stria is brighter when viewed from the retarded side of the wave-front, but the colours are more striking on the other side; the stria appears achromatic when viewed rather obliquely from the retarded side of the wave-front.

It is found that the light diffracted by the individual striæ is partially polarised. The percentage of polarisation increases with increasing angle of diffraction, the electric vector of the favoured component being *perpendicular* to the edge on the retarded side of the wave-front and parallel to it on the other side. The partial polarisation is, however, less marked on the retarded side of the wave-front, while on the other side, it is easily observed and in some cases is found to be nearly complete at an angle of diffraction of 90°. A quantitative comparison with the case of a steel razor blade shows, however, that the radiation by the laminar boundary is less perfectly polarised. Examination by a double image prism shows that the two images of the striæ may exhibit a difference in colour as well as of intensity. More detailed studies of this effect and of the elliptic polarisation to be expected when the incident light is polarised in any azimuth are, however, lacking.

The visibility of the striæ under the high powers of the microscope is a consequence of their being extremely sharp boundaries which diffract light. Under low powers, the mica striæ appear as bright lines on a dark ground when viewed under oblique illumination and as dark lines on a bright ground when seen in direct light. The diffraction of light by a stria and its visibility under the microscope are thus closely connected. The stria, seen by direct light and out of focus, exhibits the usual Fresnel pattern. If the retardation be not too great, this has a coloured centre, and the fringes on either side of it have a spacing which progressively diminishes from the centre outwards. As the focus is approached, the fringes become narrower, and the character of the pattern also alters. At critical focus the stria appears as a perfectly black line with equally spaced dark and bright fringes bordering it, these being often more numerous and distinct on the thicker side of the stria than on its thinner side. To investigate whether this pattern is influenced by the phase difference on the two sides of the edge, a monochromator may be used as the light source, and the wave-length continuously varied. The surprising and interesting observation* is then made that apart from a narrowing down of the fringes with diminishing wave-length, the nature of the pattern *as seen at critical focus* does not notably alter as the wave-length of the light is changed over the whole range of the spectrum from red to violet (see Fig. 52). On the other hand, the Fresnel patterns of the same striæ when seen out of focus show striking changes, appearing very prominently at some wave-lengths, and nearly disappearing at the other lengths, these being different again for the different

* Previously unpublished observation by the author.

striæ (see Fig. 53). It thus appears that the phase-relation of the wave-fronts on the two sides of a stria does not notably influence its microscopic aspects when seen in focus, and that the

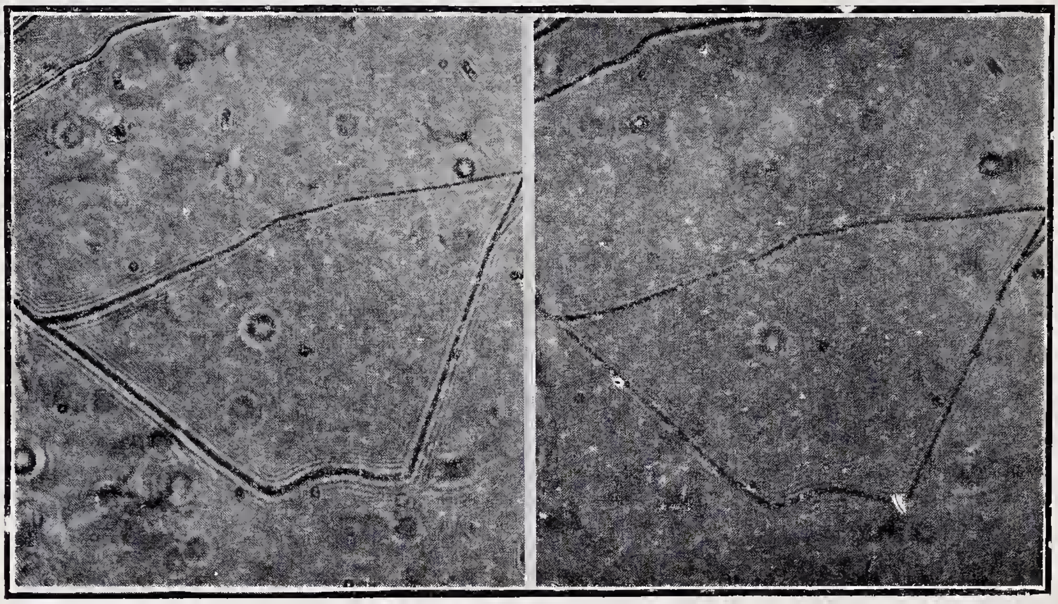

Fig. 52

5400Å 6200Å

Mica Striæ in the Microscope (in focus)

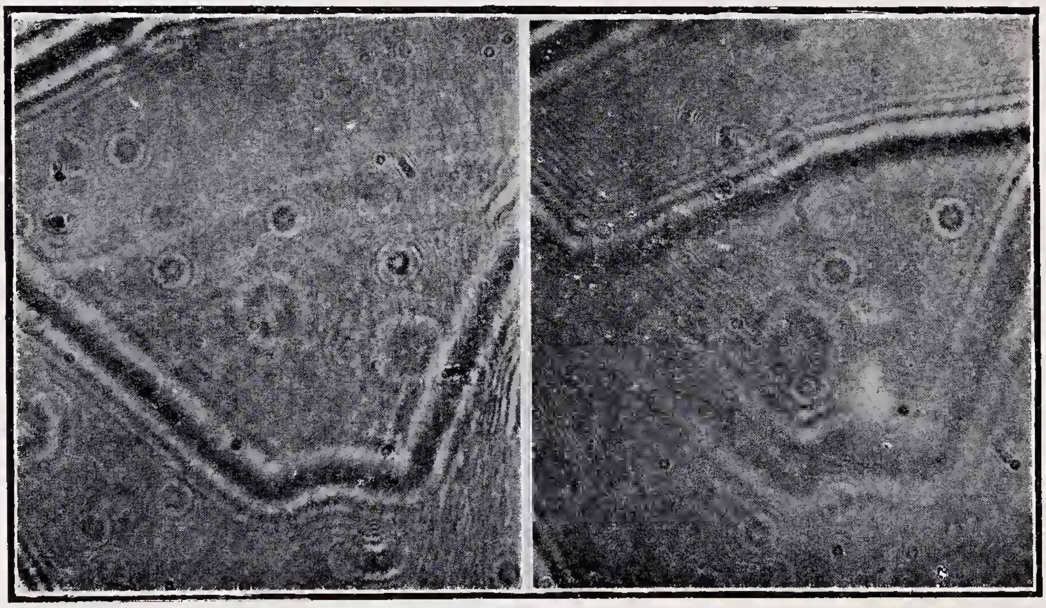

Fig. 53

5400Å 6200Å

Mica Striæ in the Microscope (out of focus) latter depends on the actual difference of thickness of the mica on the two sides of the stria and on its unresolved micro-structure. The asymmetry of the diffraction pattern at critical focus appears to be variable. In some cases, hardly any fringes are seen on the thinner side of the mica, while in others, several may be seen, though generally fewer than on the thicker side.

Very significant also are the effects noticed when the microscope is put *slightly* out of focus one way or the other. In the case in which the pattern at critical focus is strongly asymmetrical, it is found that when the microscope objective is slightly pushed towards the mica, a bright band of light forms on the thicker side of the pattern and moves into the thinner side. This effect is analogous to the well-known Becke phenomenon, and the observations indicate that it is a consequence of the unsymmetrical diffraction of light through large angles by a laminar boundary, for which direct evidence is forthcoming, as already set out. The nature of the pattern seen at critical focus also alters and in an unsymmetrical way when the incidence of the light on the mica is made oblique. The fringes crowd up towards one side of the edge, becoming very fine and numerous and remaining well-defined, while on the other side, they recede from the edge, and become fewer, broader and more diffuse. The effects are reversed when the incidence is altered from one side to the other, the fringes being better seen in either case when they are on the thicker side of the mica.

It is evident from the observed polarisation of the light diffracted through large angles, the variations of its colour and intensity and especially from the microscopic phenomena described above, that a theory of laminar diffraction which considers only the differences in phase and amplitude of the waves transmitted on either side of the boundary can, at best, give only a very imperfect account of the phenomena. By proceeding somewhat on the same lines as those adopted in the case of sharp metallic edges earlier in this lecture and considering also the waves reflected at the boundary, the position may be somewhat improved. In particular, we may get at least a general indication of the nature of the polarisation effects to be expected. Such a treatment, however, fails to explain the *asymmetry* with respect to the direction of the incident rays which is an essential and important feature in the observations. It is evident, therefore, that a satisfactory theory of laminar diffraction is, as yet, lacking.

Talbot's and Powell's Bands.—These bands are observed with white light in a spectroscope under certain conditions when one half of the aperture of the instrument is covered by a retarding plate. The theory of these bands accordingly depends on the nature of the Fraunhofer diffraction pattern due to a rectangular aperture, one half of which is covered by a retarding plate. This is evidently the same as the pattern due to the separate halves of the aperture, but modified by their mutual interference. The intensity in the pattern may be found in the usual way by integrating over the area of the aperture, and comes out (omitting a constant factor) as

$$4a^2 \frac{\sin^2 \eta}{\eta^2} \cdot \cos^2 (\delta - \eta),$$

where $\eta = \pi a \sin \theta/\lambda$, a being the half width of the aperture, θ the angle of diffraction, and 2δ the retardation produced by the plate. In the particular case when δ is zero or any multiple of π, the pattern is identical with that of the complete aperture of width $2a$. More generally, the pattern is crossed by the interference bands expressed by the factor $\cos^2(\delta - \eta)$, the values of θ for which the resulting intensity is zero being given by

$$\sin \theta = (\mu - 1)\, t/a - (n + \tfrac{1}{2})\, \lambda/a,$$

t being the thickness of the retarding plate and μ its refractive index. The angular separation of the interference bands is thus inversely proportional to the aperture. Considering the particular interference band appearing within the central fringe of the pattern, the shift of its angular position with a change of wavelength $d\lambda$ is given by

$$\lambda \frac{d\theta}{d\lambda} = -\left(\mu - 1 - \lambda \frac{d\mu}{d\lambda}\right) t/a,$$

and is thus proportional to the thickness of the plate and inversely proportional to the aperture.

This shift of the interference fringes with wave-length may be exhibited by setting the focussed pattern for white light *transversely* on the slit of a spectrograph. The interferences then appear as bands *obliquely* traversing the spectrum,* one such band being visible within the central fringe at any given wave-length (see Fig. 54). The inclination of the interference bands

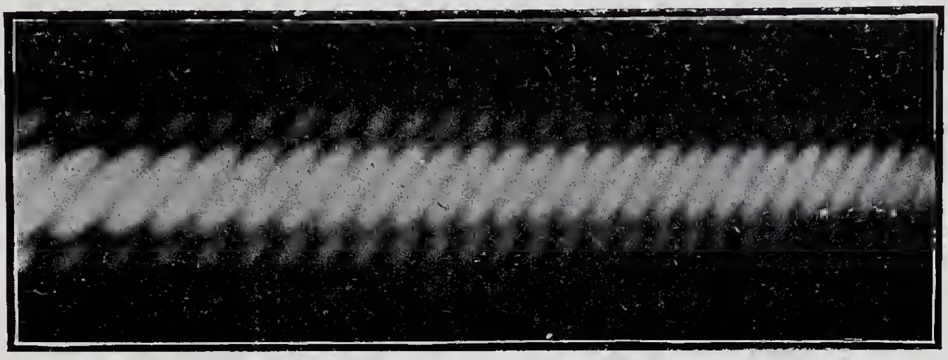

Fig. 54
Spectral Analysis of Laminar Diffraction

to the direction of the spectrum is determined by the ratio of the shift $d\theta/d\lambda$ parallel to the slit and of the shift $d\psi/d\lambda$ perpendicular to it due to the dispersion of the instrument. The separation of the successive interferences along the spectrum is determined by the quantity

$$\lambda/d\lambda = \left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right) t/\lambda = -a\frac{d\theta}{d\lambda}$$

and is thus inversely proportional to the thickness of the plate. If $\lambda/d\lambda$ exceeds the resolving power of the spectrograph, the bands would naturally cease to be observed. The visibility of the interferences produced by a thick retarding plate thus provides a test of the resolution available. It can easily be arranged that the retardation giving the diffraction pattern arises from the difference of the refractive indices of two media. For instance, a glass plate of thickness t and refractive index μ_g may be immersed in a cell of liquid of refractive index μ_l covering half the aperture; the path retardation would then be $(\mu_g - \mu_l)t$ and the shift of the interferences with wave-length would be given by

$$\lambda\frac{d\theta}{d\lambda} = -\left[\left(\mu_g - \lambda\frac{d\mu_g}{d\lambda}\right) - \left(\mu_l - \lambda\frac{d\mu_l}{d\lambda}\right)\right] t/a.$$

* N. K. Sethi, *Phys. Rev.*, 1920, 16, 519.

Thus, when the fringes given by this arrangement are set transversely on the slit of a spectrograph, whether the interference bands in the spectrum slope up or slope down, is determined by the sign of the difference between the quantities $\left(\mu-\lambda\frac{d\mu}{d\lambda}\right)$ for the two media which are their group refractive indices and not by the difference of their wave refractive indices μ Only at the particular wave-length for which the group indices for the glass and the liquid are the same, would the fringes run horizontally in the spectrum. On either side of this wave-length, the bands would slope in opposite directions (see Fig. 55).

FIG. 55
Spectral Analysis of Laminar Diffraction

If the central fringe of the diffraction pattern in white light is set on the spectrograph *parallel* to the slit and not transversely, the interference bands would now appear crossing the spectrum in the direction of the slit. Their separation along the length of the spectrum would, however, be the same as before, and their visibility would be subject to the same condition, namely, the sufficiency of the resolving power of the instrument. If now the slit of the spectrograph is opened wide, very remarkable changes would be noticed in the visibility of the interferences, depending upon which side of the aperture forming the diffraction pattern is covered by the retarding plate.* In one position of the retarding plate, the visibility of the fringes in the spectrum rapidly diminishes to zero when the slit is opened and after some feeble reappearances finally vanishes. In the other

* N. K. Sethi, *Phil. Mag.*, 1921, 41, 218.

position of the retarding plate, the slit may be opened wide, and though this necessarily diminishes the purity of the spectrum, the interference bands continue to be seen in it, and indeed in favourable cases suffer no diminution in their visibility. These results are readily understood when we consider the two directions of march of the interferences, proportional respectively to $d\theta/d\lambda$ and $d\psi/d\lambda$ which are superposed in the spectrum. In one position of the retarding plate, these movements are in the same direction. Hence, the displacements are added and at any given point of the spectrum, the maxima and minima of illumination are superposed to an increasing extent with the opening of the slit. The rapid diminution of the visibility to zero after minor reappearances follows as a necessary consequence. In the other position of the retarding plate, the two directions of march of the fringes are opposed and if they are numerically equal, compensate each other. The interferences, therefore, remain fixed in the spectrum and suffer no diminution in their visibility when the slit is opened wide.

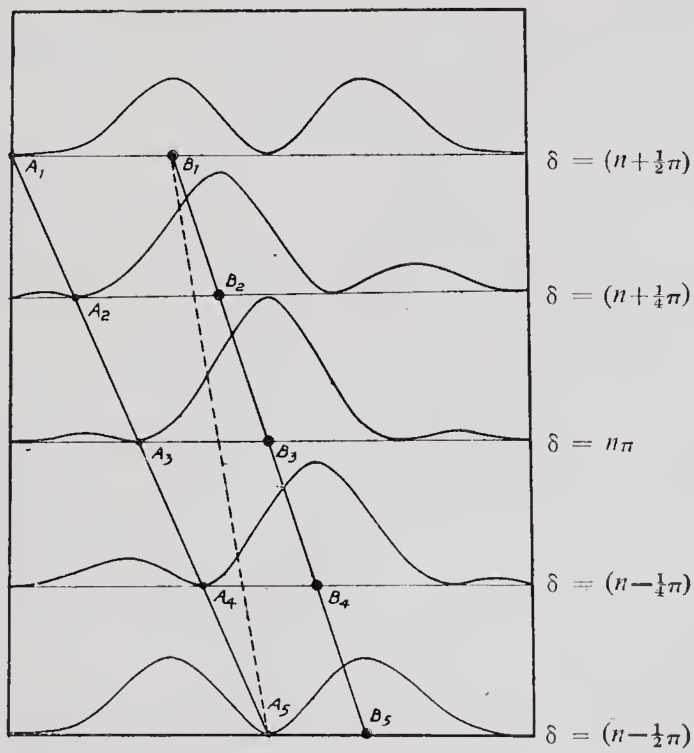

FIG. 56

Graphs of Function $\frac{\sin^2\eta}{\eta^2}\cos^2(\delta - \eta)$

The distribution of intensity in the central fringe of the pattern is shown in Fig. 56 for a series of values of δ and exhibits the lateral movement of the interferences by the slope of the line A_1 A_2 A_3 A_4 A_5 joining the zeroes of illumination in the successive curves. If, as remarked above, this slope is exactly compensated by the dispersion of the spectrograph, the interference fringes are rendered stationary and are, therefore, seen with perfect visibility. From the diagram it is also evident that if the dispersion of the spectrograph is diminished to about one-half of that required for such perfect visibility, the minimum A_5 in the fifth curve would be superposed on the maximum B_1 of the first curve, and the visibility of the fringes would, therefore, be completely destroyed. On the other hand, if the dispersion is greater than that required for perfect visibility, the straight line drawn through the minimum A_5 would slope over to the left and traverse the region outside the central fringe where the intensities are much smaller. The interferences would, therefore, continue to be visible in the spectrum though with diminished visibility.

It may be remarked that the foregoing discussion practically covers the theory of Talbot's and Powell's bands. We have only to remark that in the usual form of these experiments, the retarding plate is inserted within the spectrograph, which itself simultaneously forms the diffraction pattern and disperses it into a spectrum. The conditions for obtaining Talbot's bands under these conditions immediately follow from the foregoing theory and may be stated very simply. *The visibility of Talbot's bands is perfect when the resolving power of the spectrograph at full aperture is twice the minimum required for separating the bands. The bands are invisible when the resolving power of the instrument is only just sufficient to separate them.* The conditions for observing the bands are much more flexible in the form of the experiment discussed above in which the laminar diffraction pattern is first formed outside the instrument and then focussed on the slit of the spectrograph. The focal length of the lens which forms the pattern and its aperture which is half covered by the retarding plate are entirely at our disposal and may be quite different from those of the dispersing instrument. It is, therefore, possible to obtain what are essentially Talbot's bands with maximum visibility under almost any desired conditions.

Perhaps the most remarkable feature of Talbot's bands is that they involve an alteration of the colour sequence in the spectrum of white light.* Under favourable conditions, a spectrum traversed by the bands appears as an echelon of colour, presenting as many discrete steps as there are bands instead of the progressive change of hue normally seen in the spectrum. This effect is best exhibited by the bands when the conditions are slightly different from those required for their perfect visibility. Referring again to Fig. 56, it will be noticed that the line $B_1\ B_2\ B_3\ B_4\ B_5$ drawn through the successive maxima of intensity has a slightly greater slope than the line $A_1\ A_2\ A_3\ A_4\ A_5$. If, therefore, the dispersion of the spectrograph is about four-fifths of that required for perfect visibility, the intensity maxima in the bands come into coincidence in the spectrum. The dark bands are then not perfectly black, but the proportions of the various wave-lengths contributing to the intensity between one minimum and the next remain constant. The result is that the band exhibits a uniform hue, and when we cross from one band to the next, there is a jump in the colour. The effect is most striking when the number of colour steps in the whole spectrum is small, say 3 or 4 or 5. A spectrum of this kind may be obtained by forming the diffraction pattern with a thin plate of mica and resolving it by a prism or grating with relatively small dispersion. With the ordinary arrangements for observing Talbot's bands, the colour jumps may be effectively demonstrated even with as many as 25 or 30 bands in the spectrum, by arranging to move the latter over a slit behind which the eye is placed so as to view the surface of the dispersing prism or grating.

Before leaving this subject, attention may be drawn to the fact that the dispersive power of the retarding plate appears explicitly in the theory of Talbot's and Powell's bands. The significance of this is that, as in all interference experiments with non-homogeneous light, the observed phenomena are determined by the group velocity and not by the wave-velocity of light in the

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* N. K. Sethi, *Phil. Mag.*, 1921, 41, 211.

material media. This is very prettily illustrated with Powell's bands when the strongly dispersive mixture of benzene and carbon disulphide is employed to fill the cell in which the retarding glass plate is immersed.* The refractive index of the mixture may be steadily diminished by addition of benzene. Fairly thick plates may be used as the retardations involved depend only on the differences between glass and liquid. The point at which there is equality of refractive index between glass and liquid may be nicely judged by viewing a source of light obliquely through the edge of the plate. It is found that the bands are visible throughout the spectrum at that stage, and continue to be visible until the refractive index is further lowered and the group indices for the glass plate and the liquid mixture are equalised for some particular point in the spectrum. This corresponds to the wave-length at which the bands curve round in Fig. 55, and as it advances further into the spectrum, the bands on one side of it disappear. If, now, the position of the plate in the cell is reversed, the bands become visible in the part of the spectrum in which they were previously invisible, and *vice versa.*

It may be mentioned that spectra crossed by interferences analogous in principle to Talbot's bands and Powell's bands may be obtained when we have a succession of light beams differing in path by equal amounts (instead of only two) interfering with each other. These may be obtained by using a number of retarding plates in echelon order, or by multiple reflection between parallel surfaces as in a Fabry-Perot etalon or a Lummer-Gehrcke plate. In the latter case, it is necessary to immerse the plate in a dispersive medium.‡

Oblique Reflection and Refraction.—The surface of separation between two media differing in their properties is the seat of familiar optical phenomena, *viz.*, reflection, refraction and total reflection. These appear in the electromagnetic theory of light as consequences of the difference in dielectric constant of the two media. By considering the conditions which have to be satisfied

* N. K. Sethi, *Phys. Rev.*, 1920, 16, 519.
‡ N. K. Sethi, *Ibid.*, 1921, 18, 389.

at the boundary, which is assumed to be of unlimited area, the laws of reflection and refraction and the intensities of the reflected and refracted beams may be deduced. In the case of total reflection, the theory also leads to the conclusion that there is a superficial disturbance in the second medium. In practice, however, the surface of separation between the two media is of finite extension, and it follows that reflection, refraction and total reflection are necessarily accompanied by diffraction phenomena. We shall now proceed to consider the special features arising in these cases.

The diffraction patterns resulting from the reflection of light at a plane optical surface are easily observed.* A prism is set on the table of a spectrometer, and the slit of the collimator is viewed by reflection at one of the surfaces of the prism through the observing telescope. As the incidence of the light on the surface is made more oblique, the image of the slit broadens into diffraction pattern, the extension of which depends on the wavelength of the light employed, the width of the prism face and the angle of incidence. At moderate incidences, the pattern is indistinguishable from that of a rectilinear slit held normally in the path of a parallel beam of light. With increasing obliquity, however, the pattern progressively changes in character and becomes unsymmetrical (see Fig. 57). The fringes increase in width and at an accelerated rate as we pass from one side of the central fringe to the other. It is evident that on the side of the pattern where the fringes are narrower, there is no specifiable limit to their number, while on the side where they are broader, their number is finite. This limitation of the number of fringes on one side arises from the fact that the pattern does not extend beyond its intersection with the plane of the reflecting surface. Another noteworthy feature is that the corresponding bands on either side of the central band are of very different intensities. The fringes on the side where they are broader and fewer are much fainter than the corresponding narrow bands on the other side. This difference becomes more conspicuous when the bands compared are respectively nearer and farther from the limit of

* C. V. Raman, *Phil. Mag.*, 1906, 12, 494.

the pattern. It is also evident that on the side of the pattern remote from this limit, the successive fringes fall off in intensity more slowly than in the normal diffraction pattern of a rectangular aperture.*

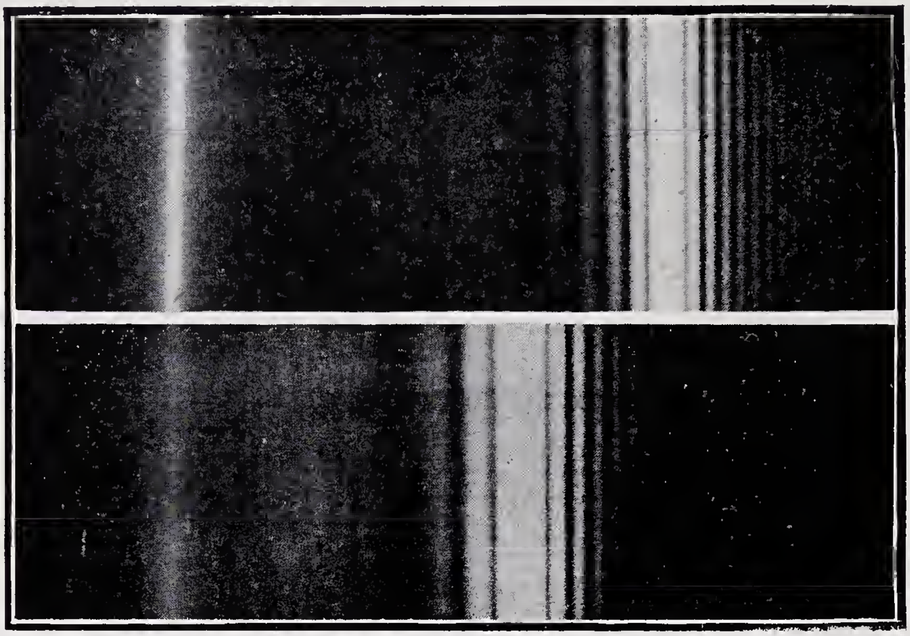


FIG. 57
Diffraction Patterns of Obliquely Reflected Light

Very similar effects are also noticed in oblique refraction at a plane surface. It is well known that an ordinary prismatic spectroscope may be adjusted to give a large dispersion by placing the prism on the table of the instrument in such a position that the light falls at nearly the critical angle of incidence on its second face and emerges in a direction almost parallel to it. There is, however, no gain of resolving power by placing the prism in this position, as the image in the focal plane of the observing telescope is greatly broadened by diffraction. The image is also strongly curved, the deviation being appreciably different for the rays which have passed in slightly different planes through the prism. Owing to the large dispersion produced by the prism, it is necessary to use monochromatic light to observe the diffraction pattern of the obliquely emergent light in the viewing telescope.

* C. V. Raman, *Phil. Mag.*, 1909, 17, 204.

Except for the curvature of the fringes and their greater width, the diffraction photographs reproduced in Fig. 58 for two different angles of emergence are very similar to the patterns for

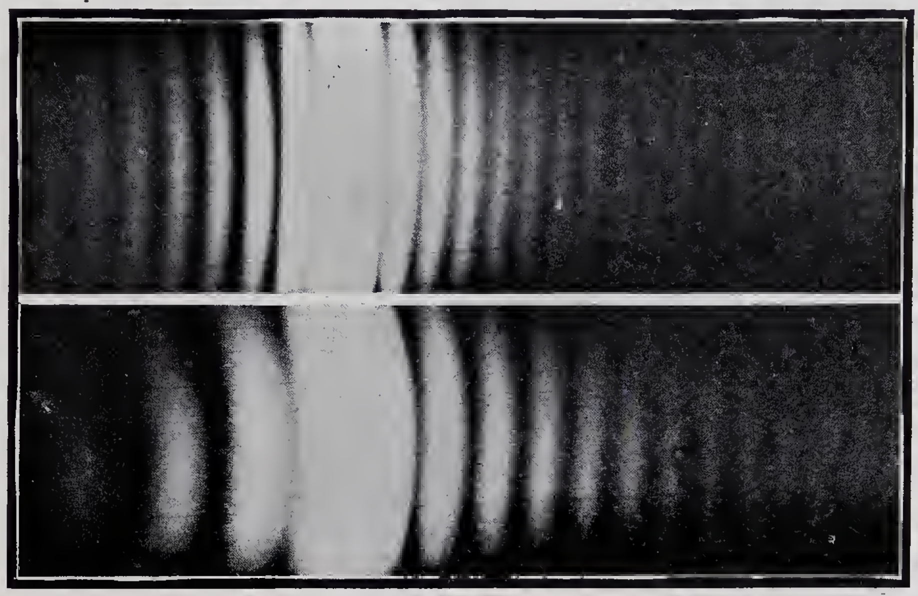


Fig. 58
Diffraction Patterns of Obliquely Refracted Light

oblique reflection discussed above and illustrated in Fig. 57. The fringes show a progressive increase in width from one side of the pattern to the other. Their number on one side is limited by the fact that the plane of the surface of the prism limits the extension of the pattern in that direction. There is also a striking difference in the intensity of the corresponding fringes on either side, the wider fringes being also the fainter. The great number of narrow fringes visible on one side of the pattern indicates that on this side they diminish in intensity more slowly than in the normal diffraction pattern of a rectangular aperture.

With exactly the same arrangements as those used in obtaining the diffraction photographs reproduced in Fig. 58, if the incidence is increased beyond the critical angle, all the fringes on one side of the pattern and the central fringe move out and disappear, but those on other side persist.* As the angle of incidence is further increased, more fringes move out of the field, but other fringes move into it, with the result that the general

* C. V. Raman, *Phil. Mag.*, 1925, 50, 812.

appearance of the pattern remains much the same except for the diminished intensity and width of the fringes. Indeed, it is clear that the diffraction phenomena at incidences less and greater than the critical angle form a continuous sequence. The slow

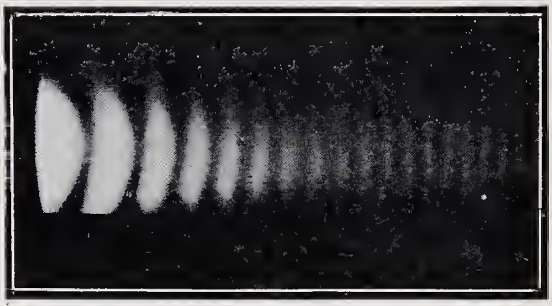


FIG. 59
Diffraction Pattern in the Second Medium in Total Reflection

diminution in the intensity of the successive fringes observed in Fig. 59 has evidently the same origin as the corresponding feature in Fig. 58.

The patterns illustrated in Figs. 57, 58 and 59 are explicable as interferences of the radiations diffracted by the edges of the surface at which reflection or refraction occurs. Alternatively, we may consider them as interferences of the secondary radiations having the elements of area of the reflecting or refracting surface as their origin. Accepting the latter view, we obtain by integration over the surface the expression for the intensity in the pattern, namely,

$$a^2/f\lambda . \cos^2 \psi . \sin^2 \xi/\xi^2$$

where

$$\xi = \pi a (\sin \psi - \sin \psi_0)/\lambda \text{ (for reflection)}$$

$$\xi = \pi a (\sin \psi - \mu \sin \psi_0)/\lambda \text{ (for refraction).}$$

ψ_0 and ψ are the angles of incidence and diffraction respectively. The zeroes of intensity appear at the values of ψ found by making $\xi = \pm \pi, \ \pm 2\pi, \ \pm 3\pi$, etc.

A remark is necessary regarding the factor $\cos^2 \psi$ appearing in the foregoing expression for the intensity in the pattern. It results from assuming that the amplitude in the hemispherical secondary waves from the elements of the surface is proportional to the cosine of the angle of diffraction, in other words, that it is greatest at the vertex of a hemisphere and zero at its base. The introduction of this obliquity factor in the law of the secondary wave is found to be necessary for an explanation of the observed distribution of intensity in the patterns. Photometric studies* have confirmed its correctness in the case of reflection‡ as well as of refraction§ at individual plane surfaces, as also at a whole series of surface elements lying in a plane and forming a diffraction grating. That the inclusion of the obliquity factor $\cos^2 \psi$ in the expression for the intensity in the pattern is necessary is also indicated by a consideration of the total energy appearing in it. To show this, we may take the case of a perfectly reflecting surface on which light is obliquely incident. The energy received by the surface is that passing through an area $a \cos \psi_0$ of the incident beam, and the same should appear in the diffraction pattern. The energy actually appearing in the latter according to our formula is

$$\int \frac{a^2}{f\lambda} \cdot \cos^2 \psi \cdot \frac{\sin^2 \xi}{\xi^2} . f . d\psi,$$

and this may be written in the form

$$\int \frac{a}{\pi} . \cos \psi_0 \cdot \frac{\cos \psi}{\cos \psi_0} \cdot \frac{\sin^2 \xi}{\xi^2} . d\xi .$$

This permits of being equated to $a \cos \psi_0$. For, the integral of $\sin^2 \xi / \xi^2$ with respect to ξ when a sufficient number of fringes is present on both sides of the pattern is π; in such an integration, we may without sensible error write ψ equal to ψ_0, its value for the central fringe which contains the largest part of the total energy. On the other hand, when some of the fringes in the part of the pattern where $\psi > \psi_0$ have disappeared, the loss of their contribution to the integral would be compensated for by the increased intensity of the fringes for which $\psi < \psi_0$.

It is worthy of remark that the diffraction of light obliquely emergent from a plane surface plays an essential role in the operation of the well-known and valuable instrument known as

* C. V. Raman, *Phil. Mag.*, 1911, 21, 618.
‡ S. K. Mitra, *Ibid.*, 1918, 35, 112.
§ B. N. Chakravarty, *Proc. Roy. Soc., A*, 1921, 99, 503.

the Lummer-Gehrcke plate. It is not unusual to find this described as an interferometer. The theory of the instrument as actually employed is, however, far from being that of a simple interference plate. In practice (see Fig. 60), the light is admitted into it through a reflecting prism cemented or optically attached to one end, and the reflection at the external surface of

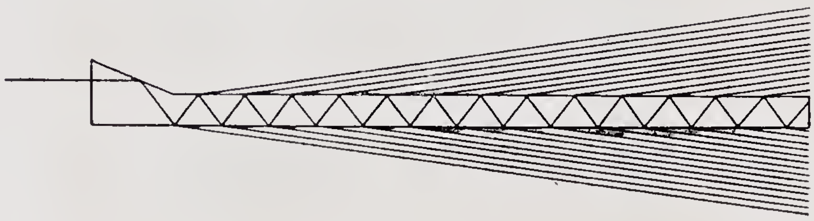


Fig. 60
Diagram of the Lummer-Gehrcke Plate

the plate is thus avoided. This makes the plate a "direct-vision" instrument and results in a large gain of illumination. As a result of this arrangement, also, we have *exactly* similar patterns on both sides of the plate, and the complementary character of the results as observed by transmitted and reflected light is no longer in evidence. In other words, *the Lummer plate functions as a diffraction grating and not as a simple interference plate.* Corresponding to a definite direction of incidence within the plate, the direction of emergence of the light may coincide with any one of several orders of interference. That this should happen is not surprising, as the individual beams emerging from the plate very obliquely are necessarily of limited aperture and are, therefore, immensely widened by diffraction. Though, in practice, an extended source of light is employed, this is not essential and the pattern may be observed even with a narrow slit, and indeed even when the light is incident within the plate at more than the critical angle. This is shown by the photographs reproduced in Fig. 61 which were obtained with the full aperture of the plate and with various incidences, using the 4358 A.U. radiations of the mercury lamp.

Fig. 61 shows that the position of the interferences remain unaffected, though the distribution of intensity amongst them alters, when the angle of incidence of the light is varied. This circumstance which enables an extended source of light to be used for observing the interferences follows as a consequence of

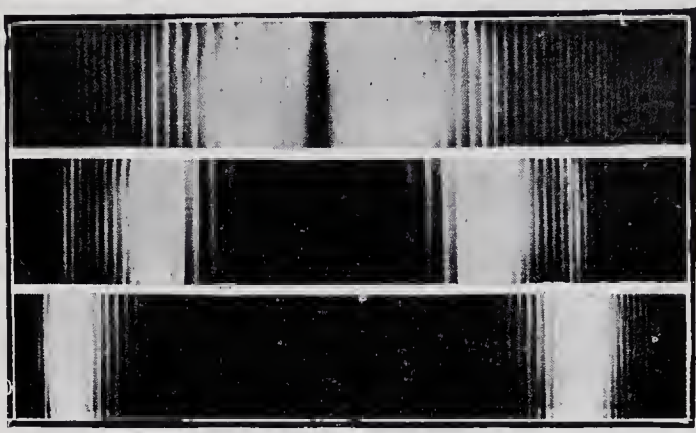


FIG. 61
Lummer Patterns at Different Incidences

the Fermat principle of stationary path. If θ and ϕ are the glancing angles of incidence and emergence of the light beams for a given order of interference, λ and μ being respectively the wave-length of the light and the refractive index of the plate, it is readily shown that $d\phi/d\lambda$ has the same value whether we regard the arrangement as an interference plate or as a diffraction grating. In the former case, we vary both θ and ϕ and maintain the geometric relation $\cos\phi = \mu \cos\theta$ between them. In the latter case, we keep θ constant and vary ϕ maintaining the path-difference between the successive diffracted beams as a constant multiple of the wave-length. In either case, we obtain the same result, namely,

$$\sin\phi \frac{d\phi}{d\lambda} = \frac{1}{\lambda \cos\theta}\left(\mu \sin^2\theta - \lambda \frac{d\mu}{d\lambda}\right).$$

The aggregate aperture of the emerging pencils is $l \sin\phi$, where l is the length of the plate; the ratio of this to the wave-length determines the angular width of the diffraction maxima and, therefore, also the resolving power of the instrument. The latter may be evaluated by writing $d\phi = \lambda/l \sin\phi$ in the preceding formula, and we thus obtain

$$\frac{\lambda}{d\lambda} = \frac{l}{\lambda \cos\theta}\left(\mu \sin^2\theta - \lambda \frac{d\mu}{d\lambda}\right).$$

The Phenomena of Total Reflection.—Total reflection was first explained on wave-principles by Huygens. Assuming that secondary wavelets issue from the surface into both media, he showed that when the incidence is beyond the critical angle, no common envelope can be drawn to the wavelets in the second medium and no resultant wave can, therefore, emerge into it. Supplementing this argument by the principle of interference, it can be rigorously proved that there should be a superficial disturbance in the second medium. It will be useful first to show this in an elementary way by the familiar method of the Fresnel zones. To mark out the form of the zones on the surface, we consider some particular point of observation in the second medium and drop a perpendicular from it on the surface. Around the foot of this perpendicular as centre, circles are drawn of which the distances from the point of observation increase successively by units of half a wave-length. Parallel and equidistant straight lines are similarly drawn perpendicular to the plane of incidence and indicating the points on the surface at which the phase of the incident plane waves differs by half a period. Both the circles and straight lines are serially numbered, and by adding these numbers at the points of intersection, the points of constant total path may be found and curves drawn through these to represent the Fresnel zones on the surface for the particular point of observation. The form of the zones thus derived and the changes in their configuration with the angle of incidence and the position of the point of observation enable us to obtain a general and comprehensive view of the case.* The zones are closed curves only when the angle of incidence is less than the critical angle; at and beyond the critical incidence, they open out and assume approximately hyperbolic forms. It is evident that there are no poles or points of stationary phase on the surface at such incidences, and the disturbance in the second medium is, therefore, a residual or diffraction effect.

Considering the disturbance at some point fairly close to the surface, it is evident that the contribution to this from distant

* C. V. Raman, *Proc. Ind. Assoc. Cult. Sci.*, 1926, 9, 271 and 330. (The second reference contains some remarks on the sharpness of the boundary of critical reflection as affected by diffraction when the aperture is limited.)

parts of the surface is insignificant. For, the obliquity factor being the cosine of the angle which the diffracted ray makes with the normal to the surface, the effect of such parts of the surface becomes vanishingly small. On the other hand, the parts of the surface near the points of observation have the maximum value of the obliquity factor, namely, unity. In this region, remarkable changes in the form of the Fresnel zones are observed if the point of observation is at or near the surface. Fig. 62 shows the

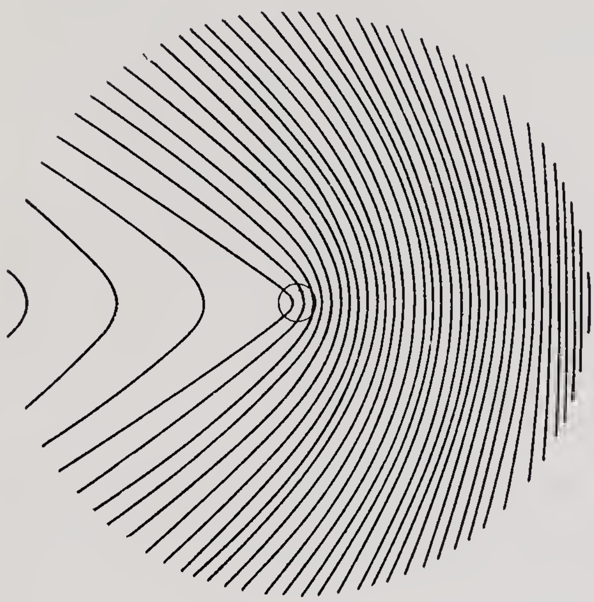


FIG. 62
Form of Fresnel Zones on Surface. Incidence at 60° (Critical Angle 45°); Point of Observation on Surface

form of the zones for a case in which the incidence is at 60°, the critical angle being 45°, while the point of observation is on the surface itself. Fig. 63 shows the zones for the same case when the point of observation is at a distance of 4 λ from the surface. A sudden discontinuity in the width of the Fresnel zones to the right and left of the origin will be noticed in Fig. 62. Accordingly, there must be a large resultant effect at this point. In Fig. 63, the discontinuity has disappeared, there being now a steady increase in the width of the zones as we pass from one side of the origin to the other. Accordingly, the effects of the Fresnel zones should cancel each other out by interference. The configuration of the Fresnel zones thus clearly indicates that there is a strong superficial disturbance in the second medium, and that this diminishes rapidly as we move away from the surface. The further we are from the critical incidence, the more quickly does this diminution occur.

FIG. 63
Form of Fresnel Zones; Incidence at 60° (Critical Angle 45°); Point of Observation 4 λ from Surface

When the point of observation is sufficiently removed from the surface, the contribution from the neighbouring parts of the surface becomes entirely negligible. On the other hand, the effects arising at the margin of the illuminated area of the surface then come into prominence. The magnitude of these effects depends on two factors, namely, the width of the uncompensated zones at the edges, and the obliquity factor. These factors work in opposite directions, since the width of the zones would be greatest when the direction of observation is parallel to the surface, while in this direction the obliquity factor actually vanishes. The most suitable way of observing the edge effects is to view them obliquely from a point entirely outside the surface, and in a direction nearly parallel to it. Both the rear and front edges then appear as fine luminous lines and are seen to be of equal intensity. The interference of the edge radiations gives rise to a diffraction pattern which can be recorded photographically with sufficiently long exposures* (see Fig. 59).

* C. V. Raman, *Phil. Mag.*, 1925, 50, 812.

Returning to the effects observed at a point P near the surface, we may derive an expression for the superficial wave in the second medium. The perpendicular drawn from P to the surface is taken as z-axis, its foot as the origin, and the plane of incidence of the light as the XZ plane. The light vector in the first medium along the surface may be assumed to be of the form

$$A\,\sigma_0 \cdot \cos\,(Q_0 - 2\pi x \sin\psi_0/\lambda), \text{ or simply } A\,\sigma_0 . \cos Q$$

for brevity, σ_0 being an undetermined constant. The surface is divided up into elements of area $\rho d\rho d\phi$ in polar co-ordinates, r the distance of the element from P being $(z^2+\rho^2)^{\frac{1}{2}}$. The effect of the secondary wave from such an element at P is

$$A\sigma_0 \cdot \frac{\rho d\rho d\phi}{\lambda r} \cdot \cos\,(Q - 2\pi\mu \cdot \rho \sin\psi_0 \cos\phi/\lambda - 2\pi r/\lambda).$$

The obliquity factor is taken as unity, since the effect at P is mainly due to elements on the surface near the origin. Integrating with respect to ϕ between the limits 0 and 2π, we obtain the resultant effect at P as

$$A\sigma_0\,(C \cos Q + S \sin Q)$$

where C and S respectively stand for

$$C = \frac{2\pi}{\lambda} \int_0^\infty J_0\left(\frac{2\pi\mu\ \rho \sin\psi_0}{\lambda}\right) \cos \frac{2\pi r}{\lambda}\,\frac{\rho d\rho}{r}$$

$$S = \frac{2\pi}{\lambda} \int_0^\infty J_0\left(\frac{2\pi\ \mu\ \rho \sin\psi_0}{\lambda}\right) \sin \frac{2\pi\ r}{\lambda}\,\frac{\rho d\rho}{r}$$

These are well-known integrals.* When $\mu \sin\phi_0 > 1$, S vanishes and the effect at P reduces‡ to

$$A\sigma_0\,(\mu^2 \sin^2\psi_0 - 1)^{-\frac{1}{2}} \cos Q \cdot e^{-2\pi z\sqrt{\mu^2 \sin^2\psi_0 - 1}/\lambda}$$

$$= A\sigma \cos Q \cdot e^{-2\pi z\sqrt{\mu^2 \sin^2\psi_0 - 1}/\lambda}$$

thus appearing as a superficial wave $A\,\sigma \cos Q$ at the surface and travelling along the x-axis, the amplitude of which decreases exponentially with the distance z from the surface. The

* Bateman's *Electrical and Optical Wave-Motion*, p. 72; and Watson's *Treatise on Bessel Functions*, 1922, p. 416.

‡ C. V. Raman, *Trans. Opt. Soc.*, 1927, 28, 149.

disturbance at the surface in the first medium is due jointly to the incident and totally reflected waves. If these are individually

$$A \cos (Q - \delta/2) \text{ and } A \cos (Q + \delta/2),$$

their resultant is $2 A \cos \delta/2 \cos Q$. The continuity of the disturbance at the surface parallel to the y-axis requires that $\sigma = 2 \cos \delta/2$, thus showing that the amplitude σ of the superficial wave and the phase change δ in total reflection are closely connected with each other. To evaluate σ and δ, we require a second relation. This is obtained by considering the continuity of the disturbance parallel to the x-axis on the two sides of the boundary. If the electric vector in the incident waves is parallel to the y-axis, then the magnetic vector H_x, *i.e.*, $\partial E_y/\partial z$ must be the same on the two sides of the boundary. If, similarly, the magnetic vector in the incident waves is parallel to the y-axis, then the electric force E_x must be the same on the two sides; in other words

$$\left(\frac{\partial\, Hy}{\partial z}\right)_1 = \mu^2 \left(\frac{\partial\, Hy}{\partial z}\right)_2$$

Applying these conditions, the values of σ and δ in these two cases are respectively found to be

$$\sigma_s{}^2 = \frac{4\,\mu^2 \cos^2 \psi_0}{\mu^2 - 1}, \quad \tan \tfrac{1}{2}\,\delta_s = \frac{\sqrt{\mu^2 \sin^2 \psi_0 - 1}}{\mu \cos \psi_0}$$

$$\sigma_p{}^2 = \frac{4 \cos^2 \psi_0}{(1-\mu^2) + (\mu^4 - 1) \sin^2 \psi_0}, \quad \tan \tfrac{1}{2}\,\delta_p = \frac{\sqrt{\mu^4 \sin^2 \psi_0 - \mu^2}}{\cos \psi_0}$$

It is readily verified that at the critical incidence, $\delta_s = \delta_p = 0$ and $\sigma_s = \sigma_p = 2$. In other words, the amplitude of the superficial disturbance which is then a maximum is the arithmetic sum of the amplitudes in the incident and reflected waves. At grazing incidence, $\delta_s = \delta_p = \pi$, and $\sigma_s = \sigma_p = 0$. In other words, the surface is a nodal plane and the superficial wave vanishes. At intermediate incidences, δ_p is greater than δ_s and this results, when the incident light is plane polarised in an azimuth inclined to the plane of incidence, in the reflected light being elliptically polarised.

The existence of a superficial wave in the second medium may be demonstrated in several ways. A direct method which has the advantage of enabling us to determine the distribution of intensity in depth as well as direction of energy flow and the state of polarisation, is to use the well-known property possessed by a sharp metallic edge of diffracting a stream of radiation falling upon it.* A fresh safety razor-blade is held normal to the surface of a totally reflecting prism and with its edge exactly parallel to it. A fine slow-motion of the kind provided in interferometers enables the blade to be moved forward or backward by fractions of a wave-length. The razor edge is viewed through a microscope focussed on it. If the axis of the microscope is in the plane of incidence of the light, and the razor-blade is perpendicular to it, the edge when slowly advanced to within a very small distance of the surface, becomes visible as a fine luminous line. The intensity is greatest when the axis of the microscope is as nearly as possible parallel to the surface. The distance from the surface within which the luminosity of the edge is perceptible is a measure of the thickness of the superficial disturbance. It is known that a diffracting edge is only luminous when seen along the surface of a cone having the edge as axis and the ray incident on it as a generating line. Hence the observations indicate that the direction of energy-flow in the superficial disturbance is in the plane of incidence and parallel to the surface. When the incidence is just at the critical angle, the intensity of the superficial wave is found to be a maximum and comparable with that of the incident and reflected waves. As the incidence is increased, the intensity falls off very quickly. The decrease of intensity with increasing distance from the surface is also rapid. When the incidence is not much greater than the critical angle, say about 50° for glass, the luminosity is perceptible when the edge is within a wave-length or so from the surface. For larger angles of incidence, the decrease is much more rapid, and the luminosity is perceptible only when the edge is practically in actual contact with the prism. When the incident light is polarised, the edge radiation is found to be partially polarised, the stronger component of the electric vector being perpendicular to the edge. This is in accord with the theoretical expectations.

* C. V. Raman, *Trans. Opt. Soc.*, 1927, 28, 149.

Diffraction of Light by Curved Surfaces.—A convex rounded edge on which light is incident reflects the light falling upon it, and the superposition of this on the exterior diffraction increases its apparent intensity, and makes it perceptible over larger angles. On the other hand, the light bending into the region of shadow is diminished in intensity by reason of the curvature of the edge and its visibility is restricted to smaller angles. This effect may be observed with edges having a curvature which may lie within wide limits and is expressible in microns or centimetres or even metres. The angles involved and the magnitude of the effects would, however, naturally depend on the radius of curvature. A typical experiment demonstrating the effect under consideration is the comparison of the intensities of the bright spots seen at the centre of a shadow of a circular disk and of a spherical ball of the same radius.* To make the comparison significant, the disk should have a sharp edge, while the ball should be an accurately made sphere with a highly polished surface. The ball and disk may be set side by side and held in the path of a beam of light from a point-source of light. The spots at the centre of their respective shadows may then be directly viewed and their intensities photometrically compared.

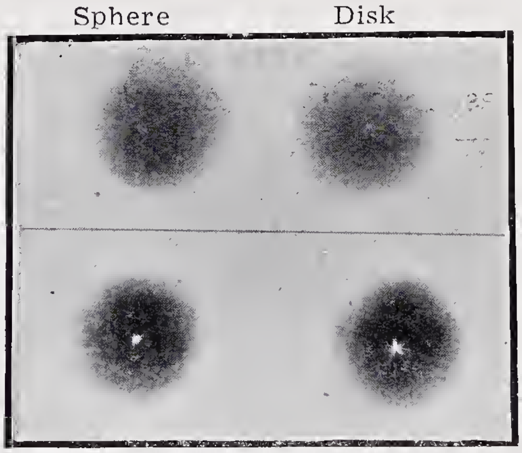


FIG. 64
Diffraction by Sphere and Disk of Equal Diameter (1·5 cm.) and at (*a*) 40 cm., and (*b*) 120 cm. distances from them

A great difference is noticed in the brightness of the two spots. This difference increases as we approach the objects

* C. V. Raman and K. S. Krishnan, *Proc. Phys. Soc.*, 1926, 38, 350.

(Fig. 64); the intensity of the spot for the sphere falls off more quickly than for the disk, becoming a very small fraction of it. At a distance of 120 centimetres, the difference of the intensities is still perceptible, while at greater distances, the two spots tend gradually to approach equality of brightness.

The diffraction of light by convex cylindrical edges similarly shows interesting features. If the radius of curvature of the edge is of the order of a few centimetres, the phenomena in the vicinity of the edge may be conveniently studied with a distant slit parallel to the cylinder as the source of light and a microscope for viewing the fringes.* The effects are, however, more striking when a strip of mirror glass, 3 centimetres wide and 75 centimetres long, is bent into a cylindrical shape of large radius of curvature. The diffraction fringes produced by it can be seen directly on a screen or viewed with an ordinary magnifier.‡ The general nature of the case will be evident when we consider the rays of light which pass by the cylinder without meeting its surface and those which fall on it and are reflected. These form the two branches of a cusped wave-front which is fully developed at the edge of the cylinder grazed by the incident rays. The two branches interfere, giving fringes parallel to the edge, their number and visibility being greatest when the plane of observation is that of the edge itself. As we move away from this plane, the intensity of the reflected rays diminishes more quickly on account of their divergence. The visibility of the fringes, therefore, falls off until, finally, they are scarcely distinguishable from the usual type of diffraction bands along the boundary of the shadow of a straight edge. The law of the spacing of these fringes may be readily deduced. If a is the radius of the cylinder, and d the distance of the plane of observation from its edge, the distance x of the maxima and minima of the illumination from the edge of the geometric shadow may be found by eliminating ϵ from the two equations

$$x = 2\epsilon d + 3\epsilon^2 a/2 \text{ and } n\lambda = 4\epsilon^2 d + 4\epsilon^3 a.$$

* N. Basu, *Phil. Mag.*, 1918, 35, 79.

‡ T, K. Chinmayanandam, *Ibid.*, 1919, 37, 9. The same procedure may be used for observing the optical analogue of the whispering gallery effect with a concave surface.

When d is much larger than a, we have $x = \sqrt{n\lambda d}$, and the positions of the fringes differ only slightly from those in the diffraction pattern due to a straight edge for which we have $x = \sqrt{(n-\frac{1}{4})\lambda d}$. Indeed except in the vicinity of the cylinder, the spacing of the fringes is scarcely different from that due to a sharp straight edge, the principal difference being in the greater number and the visibility of the fringes. Phenomena of the same nature are also observed in the exterior diffraction by reflecting obstacles of other forms having a convex surface, *e.g.*, cones, spheres or ellipsoids.*

The rapidly diminishing intensity of the light entering the region of shadow and its restriction to smaller angles with increased radius of curvature of the surface are facts of observation which require explanation. Some light is thrown on the matter by a consideration of the facts regarding the diffraction of light by metallic screens dealt with earlier in this lecture. An examination of the formulæ shows that the nature of the results to be expected is greatly influenced by the angle of incidence of the light on the surface of the screen.‡ When the incidence is sufficiently oblique, both components of the electric vector in the diffracted rays parallel to the surface of the screen become of equal intensity, and are quite small. This indication of theory is in accord with observation,§ and may be expected to be true for all reflecting surfaces whether metallic or not. In our present problem, we are concerned with the diffraction of light which is incident very obliquely or actually grazes the surface of the obstacle. Hence, unless the radius of curvature of the obstacle is very small, the polarisation effects would be negligible and both components of the diffracted light would be weak along its surface. The greater the radius of curvature, the longer the arc of the surface which the diffracted ray has to graze before it can emerge at any desired angle. Hence, the attenuation of the diffracted light must increase rapidly with the radius of curvature. The problem here considered is evidently analogous to that of the bending of electric waves around the surface of the

* A. B. Datta, *Bull. Cal. Math. Soc.*, 1922.
‡ C. V. Raman and K. S. Krishnan, *Proc. Roy. Soc.*, 1927, **116**, 254.
§ S. K. Mitra, *Phil. Mag.*, 1919, 37, 50.

earth. The attenuation factor for the amplitude in this case* is $e^{-0\cdot 70(2\pi a/\lambda)^{\frac{1}{3}}\cdot\theta}$ where a is the radius of the earth, λ the wave-length and θ the angle which the waves have to creep round. The formula indicates a rapid fall in the intensity of the radiation as it bends round, if the radius of curvature of the edge is large compared with the wave-length, and this is in accord with actual experience in the optical problem.

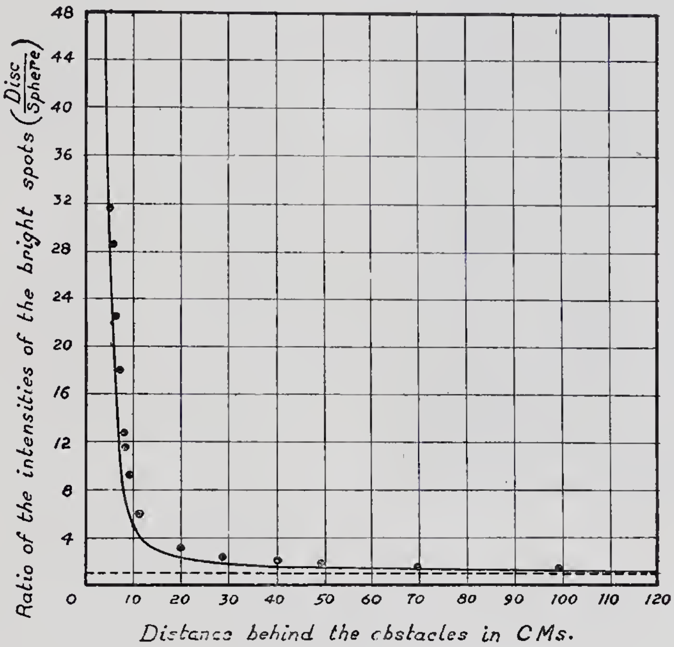


FIG. 65
Comparison of Theoretical Attenuation Factor and Experimental Data

The measurements‡ made of the relative intensity of the bright spots in the shadow of a sphere and a disk of equal diameter at different distances along the axis enable a quantitative test to be made of the attenuation formula quoted above. The arc over which the diffracted waves have to creep may be taken as zero for the sharp-edged disk. For the sphere, it is the arc on the surface between the circles of contact with the tangent cones drawn to it respectively from the point-source of light and from the point of observation. Fig. 65 shows the theoretical attenuation curve for this case, the experimental data being indicated by dots. The general agreement leaves little doubt that the explanation of the facts which has been put forward is on the right lines.

* Riemann-Weber, *Differential-Gleichungen der Physik.*, 1927, 2, 594.
‡ C. V. Raman and K. S. Krishnan, *Proc. Phys. Soc.*, 1926, 38, 350.

LECTURE III

CORONÆ, HALOES AND GLORIES

IN the present lecture, we shall consider the phenomena which arise from the diffraction of light simultaneously by a great many particles or obstacles, the size of these being sufficiently large to permit of an elementary approach to their explanation. Many such phenomena are known, and it is of advantage to consider them together in a general survey, so that the common principles underlying all such cases may be brought into relief. The optical character of the particles, their size, shape and number, the manner in which they are disposed and orientated in space, and the particular circumstances of observation may all influence the results. Included within the survey are some natural phenomena which may be observed in the earth's atmosphere when particles of water or ice are present in it and are suitably illuminated by the rays of the sun or the moon.

Diffraction by a Cloud of Particles.—Secondary radiations derived from the same primary source, and therefore having specifiable phase-relations with it and with each other, would evidently be capable of interference. Hence, when a cloud of particles is present in a light-field and the radiations diffracted by the individual particles are superposed at any given point of observation, interferences would arise. Their character would be determined by the phase-differences, in other words, by the optical paths traversed from the original source to the individual diffracting particles and thence to the point of observation. Considering first a case in which the line joining the primary light source with the point of observation passes *through* a cloud of particles, it is evident that the optical paths would differ infinitely little for all particles lying on this line or in its immediate vicinity. On the other hand, the optical path would alter in a rapidly increasing measure with the actual position of the particle as it lies further and further away from this line. Thus, in general, *except along the direction of propagation of the light rays from the original source*, the distribution of the individual diffracting particles in space is a controlling factor in determining the optical effect produced by a cloud of such particles.

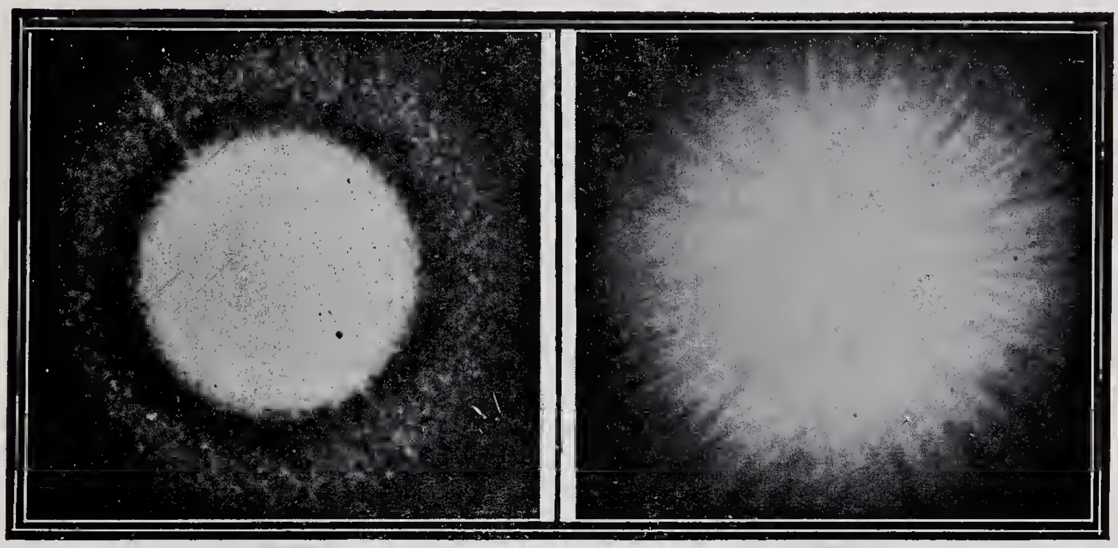
Considering the effects produced by the cloud in any direction other than that of the primary rays, we shall assume that the particles are distributed at random and execute *rapid uncorrelated movements* within the cloud. It is obvious that in such circumstances, the interferences between the effects of the individual particles would be unobservable. We may then assume the observed intensity in the field to be a summation of the intensities of the individual effects. If all the n particles were similar and produced similar effects at any point of the field, the total observed intensity would be n times the effect of an individual particle. On the other hand, *if the particles occupied stationary positions* within the cloud, the situation would be entirely different. However numerous the particles might be, and howsoever they might be distributed within the cloud, the phase-relations between them would be determinate, and hence the interferences between the individual effects should be observable. We have to evaluate the result of such interferences to find the optical effect due to the entire cloud of particles.

The problem which thus arises of finding the effect of n superposed radiations of equal amplitude but of differing phase may be dealt with graphically by means of a two-dimensional diagram. Choosing a given point O as origin, we draw a set of *n radii vectores* of equal length A representing the amplitudes of the n superposed radiations; their relative phases would be given by the angles which these make with each other. It is evident that the resultant obtained by the summation of the vectors so drawn would depend on the manner in which the terminal points of the radii are distributed around the circle on which they lie. If, for example, the phases are all identical, the vectors would all be superposed, and the resultant amplitude would be nA and the resultant intensity n^2A^2. If, on the other hand, the n vectors divide the circle into n equal arcs, the resultant amplitude and intensity would both be zero. If now, we consider the case in which the n phases are distributed at random, it is obviously impossible to specify what either the amplitude or the phase of the resultant would be *in any particular trial.* The diagram, however, gives some indications of a general character regarding what we may expect to find. If the number n be sufficiently large, the most probable location of the points on the circle in a random distribution would evidently be a sensibly uniform one. Hence, the most probable resultant intensity would be the same as for a perfectly regular distribution, namely, zero. It is evident also that the resultant intensity averaged over a large number of trials would be nA^2. This follows immediately, if we suppose that the phases vary continuously and rapidly with time, so that the n intensities, each of which is A^2, become additive.

Thus, *for a random distribution of phases, the most probable resultant intensity is zero, while the average intensity in a large number of trials would be nA^2.* For a more complete description of the case, we have to find an expression for the probability that the intensity has a specified value I in different trials. It is easily verified that this is given by the exponential probability formula

$$dW = \exp(-I/nA^2) \cdot d(I/nA^2).$$

The formula agrees with what the graphical treatment suggests; it shows that the probability is a maximum for zero intensity and that it diminishes continuously and ultimately vanishes with increasing values of the intensity. Further, the integration of dW over all possible values of I gives unity as it should, and the average intensity found by integrating IdW over all possible values of I is nA^2, as already found. Hence, the formula $dW = e^{-f}\, df$ correctly gives the probability of any given value of the observed intensity expressed as a fraction f of the average intensity. We note also that the formula agrees with that given by a detailed consideration of the problem on the basis of the general theory of probability.* It is important to notice that the chance of finding any particular resultant intensity decreases continuously as it increases, and that the average intensity is very far indeed from being the most probable intensity. Indeed, the average is determined entirely by the cases in which the resultant intensity is greater than the most probable value which is zero.



(*a*) (*b*)

Fig. 66

Diffraction Corona due to Lycopodium Spores showing (*a*) Granular Structure in Monochromatic Light, and (*b*) Radial Streaks in White Light

A point source of monochromatic light viewed through a cloud of particles would appear surrounded by a corona or halo due to diffraction by the particles. The radiations diffracted by the particles and reaching the retina of the eye and focussed thereon are superposed and would thus be capable of interfering with each other. The foregoing discussion shows that if the particles are all similar and are disposed at random in space, the intensity in the corona would only *statistically* be a summation of the intensities of the diffraction patterns produced by the individual particles. While the general features of the pattern

* Rayleigh, I., *Phil. Mag.*, 1880, 10, 73, Sc. papers, Vol. I, p. 491.

due to each separate particle would be recognisable in the aggregate effect, the latter is essentially different in detail. Instead of a continuous distribution of intensity, we have a violently fluctuating one which, in general terms, may be described as a dark field on which appear a great many points of illumination irregularly distributed and of varying brightness. The illumination at such points arises from the accidental agreements of phase of the effects of the diffracting particles, while the dark field results from the general cancellation of their effects by mutual interference. *Each such point in the corona exhibiting an observable intensity is, therefore, essentially an optical image of the original source produced by the entire cloud of particles functioning as a randomly distributed set of secondary sources of light.**

As we shall show later, the theoretical conclusions set out above are fully supported by the experimental results (see Fig. 66 *a*). It is important to remark that, in practice, cases may also arise in which the diffracting particles are not distributed at random in space. The distribution may either present a closer approach to uniformity, or may tend in the opposite direction, the individual particles clustering together to form large groups. The optical effects would in either case differ from those observed with a random spacing of the particles. In the limiting case of a perfectly uniform distribution, the particles would in effect constitute a diffraction grating. We would then get sharply-defined and intense diffraction spectra located at regular intervals in a dark field. The transitional cases, where the distribution of the particles in space is neither completely random nor completely uniform, are of particular interest. The phenomena observed in all such cases may be included under the general descriptive term of "diffraction haloes", the expression "corona" being reserved for the case of randomly spaced particles. As examples of such haloes, we may turn to Fig. 23 on page 51 of the second lecture, in which the effect of viewing a source of light through a thin piece of mother-of-pearl was illustrated. As was remarked on page 51, the size and shape of the crystallites of

* G. N. Ramachandran, *Proc. Ind. Acad. Sci.*, A, 1943, 18, 190.

aragonite, and their spacing and orientation within the mother-of-pearl determine the character of these haloes, and as will be evident from the three examples reproduced, these features and the resulting haloes are very different in the three great classes of mollusca. We shall meet with other cases of the production of diffraction haloes later in the present lecture.

Coronæ due to Water Droplets.—The well-known coronæ or disks of light with marginal coloured rings seen surrounding the sun or the moon when viewed through thin clouds are amongst the most familiar phenomena of meteorological optics. What we see in such cases is evidently the cloud itself which becomes visible by reason of the light incident on it and diffracted through various angles by the particles of which it is composed. The optical character of the phenomena, as well as the form and level of the clouds exhibiting them, make it clear that the coronæ with vividly coloured rings owe their origin to minute spherical droplets of water contained in the clouds. Thin clouds consisting of small particles of crystalline ice do exhibit observable disks surrounding the sun or moon when seen through them. But these are usually of smaller size and have a quite different and characteristic distribution of intensity. They are also much less vividly coloured than the coronæ arising from water clouds. Indeed, a cirrus haze can just as readily be recognised by the diffuse illumination observed in the vicinity of the sun or the moon as by the familiar halo due to refraction by the ice-crystals seen at an angle of 22° from the luminary. It should be remarked also that clouds do sometimes display marked iridescence in circumstances which indicate that their temperature must be well below the freezing point of water. Such iridescence is often observed at quite large angles with the sun, though, of course, a complete corona is not usually then seen. Whether such iridescent clouds consist of crystalline particles of ice is a debatable question. The vividness of the colours suggests that the particles are probably supercooled droplets of water, or possibly even an amorphous form of solidified water. The retention of an amorphous structure and of non-crystalline shape by droplets of water when supercooled is a well-established fact of observation under laboratory conditions, and it is permissible, therefore, to suppose that it can also occur in nature.

Coronæ can also be artificially produced and observed in the laboratory over a wide range of droplet size, and they are actually more striking than the coronæ seen in nature, the colours of which are somewhat diluted by the finite angular dimensions of the sun or the moon. As is well known, a sudden expansion of moist air, if of sufficient magnitude, results in the formation of a cloud consisting of minute droplets of water. The condensation usually occurs around "nucleii" of some sort, and the number of droplets formed and therefore also their size depends on the number of such nuclei present. The size of the droplets as indicated by their rate of free fall, as also by the optical effects which we shall presently consider exhibits a remarkable uniformity. It may be regulated within wide limits by varying the amount of the expansion and the number of nucleii present. Very beautiful and interesting effects are observed when such clouds are viewed

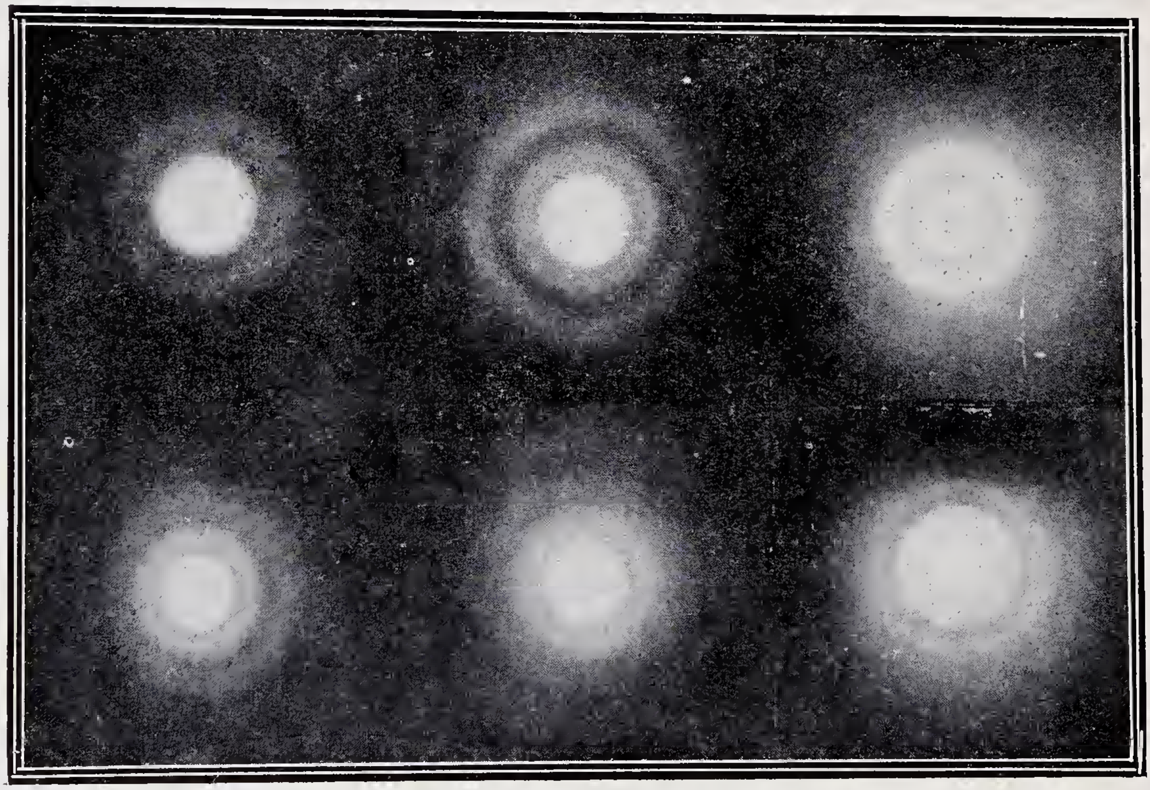


FIG. 67

Coronæ due to Water Droplets of different sizes under strong illumination, or if a bright source of light is seen through such a cloud. By using an electric arc as the source of light and projecting an image of it as seen through the cloud chamber on the screen, the coronæ due to water droplets can be shown as a beautiful lecture demonstration. Using a red glass as a monochromatising filter, or with a quartz mercury lamp and single ray filter, the coronæ may be readily photographed.

The particles of water in a cloud are very numerous and yet not so numerous as to occupy an appreciable fraction of the volume of the air. They are obviously distributed at random in the space, and presumably execute small irregular movements. It follows that though the droplets are all illuminated by the same original source of light, we may nevertheless regard them as practically independent sources of diffracted radiation. The justification for this is that the phases of the diffracted radiations in any assigned direction from the different droplets are totally unrelated. An exception must, however, be made in considering the rays diffracted in the same direction as the rays incident on the drops; for, in this direction the optical paths for all the drops are identical, and hence their amplitudes must be added to find their resultant effect. In all other directions, we may add the intensities of the diffracted radiations from the drops and expect the results to be in accord with the facts.

The appearance of the coronæ in the experiments is found to vary in a remarkable manner with the size of the droplets. The central disk of the corona as seen with the finest droplets is not white but shows vivid colour varying with their size; as the drop size is altered progressively, there is a recognisable cycle of changes in the colour observed. The sequence of changes observed with increasing size of the droplets is not a mere progressive diminution in the angular diameter of the corona as seen in monochromatic light. A periodic alteration in the diameter and intensity of the coronal disk is noticeable, while from the published photographs* it is evident that the relative intensities and positions of the outer rings vary notably when the drop size is altered.

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* M. N. Mitra, *Ind. Jour. Phys.*, 1928, 3, 175. The photographs reproduced in Fig. 67 are due to Mr. H. Ramachandran.

The experimental facts thus compel us to reject the usual explanation of coronæ which is based on the assumption that the droplets may be regarded as opaque spheres. The starting point for a more satisfactory theory is a consideration of the phase-changes resulting from the passage of plane waves through a transparent sphere of liquid. In the limiting case when the refractive index of the liquid μ is only slightly greater than unity, the waves pass through the sphere without any change of amplitude but with a change of phase $\xi \cos \epsilon$ where ξ is $4\pi a(\mu-1)/\lambda$; a is the radius of the sphere and ϵ is half the supplement of the angle subtended at the centre by the path inside the sphere, being zero for a ray passing centrally and $\pi/2$ for a marginal ray grazing the surface. The wave-front on emergence would thus exhibit *a dimple* having the same radius as the drop and a depth equal to the maximum retardation it produces. If $(\mu-1)$ be not small, this simple picture would not be accurate, as the wave-front on emergence from the drop would exhibit both amplitude and phase changes. We may, however, adopt it as the basis for an approximate theory which, though it could scarcely be expected to give a complete account of the facts, should nevertheless go far towards doing so.

If the dimples in the wave-front be removed, and the resulting holes filled up, the diffracted radiations would disappear. It follows that the effect of a drop may be found by *subtracting* from the optical effect of the dimple in the wave-front, the effect produced by plane waves of light passing through a circular aperture of the same radius in an opaque screen. The relation between the amplitude and phases of the two effects which are thus superposed determines the observed phenomena, and it is evident that the interference between them is responsible for the observed cycle of changes in the appearance of the corona with increasing drop size.

The detailed calculations are made on much the same lines as for a simple circular aperture. Besides the phase-change $\xi \cos \epsilon$, we have also to consider the phase-difference between the different parts of the wave-front introduced by the observation of their resultant at a great distance d and at an angle β with the incident rays. This may be written as $\eta \sin \epsilon \cos \alpha$, where ϵ is the angle already introduced, and α is the azimuthal angle defining the position of an element of area, *viz.*, $a^2 \sin \epsilon \cos \epsilon \, d\epsilon \, d\alpha$, in the wave-front emerging from the drop. η stands for $2\pi a \sin \beta/\lambda$. The disturbance in the direction β due to the light which has traversed the drop is given by the integral

$$\int_0^{\pi/2}\int_0^{2\pi} \frac{a^2}{\lambda d} \sin (Z - \xi \cos \epsilon + \eta \sin \epsilon \cos \alpha) \sin \epsilon \cos \epsilon \, d\epsilon \, d\alpha.$$

On integration with respect to α, this yields

$$\frac{2\pi a^2}{\lambda d} \int_0^{\pi/2} J_0 (\eta \sin \epsilon) \sin (Z - \xi \cos \epsilon) \cos \epsilon \sin \epsilon \, d\epsilon.$$

If we put $\mu = 1$, ξ vanishes, and the integral reduces, as it should, to the effect of a simple circular aperture of radius a, namely,

$$\frac{2\pi a^2}{\lambda d} \sin Z \cdot \frac{J_1 (\eta)}{\eta},$$

and, as already remarked, the contribution of the drop to the corona is found by deducting this from the foregoing integral.

In the exact forward direction, β is zero and η vanishes. The foregoing integral can then be completely evaluated, and after the deduction indicated is made, it gives for the amplitude the expression*

$$\frac{2\pi a^2}{\lambda d} \sin Z \left(\frac{\cos \xi}{\xi^2} + \frac{\sin \xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right) + \frac{2\pi a^2}{\lambda d} \cos Z \left(\frac{\cos \xi}{\xi} - \frac{\sin \xi}{\xi^2}\right).$$

The intensity of forward scattering is thus

$$I_f = \frac{4\pi^2 a^4}{\lambda^2 d^2} F (\xi),$$

where

$$F (\xi) = \left[\left(\frac{\cos \xi}{\xi^3} + \frac{\sin \xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right)^2 + \left(\frac{\cos \xi}{\xi} - \frac{\sin \xi}{\xi^2}\right)^2\right].$$

The discussion hereafter follows, in the main, two papers by G. N. Ramachandran.§ $F(\xi)$ is plotted against ξ in Fig. 68 exhibiting the manner in which the forward intensity varies with the

* T. A. S. Balakrishnan, *Proc. Ind. Acad. Sci.*, A, 1941, 13, 188.
§ G. N. Ramachandran, *Ibid.*, 1943, 17, 171 and 202.

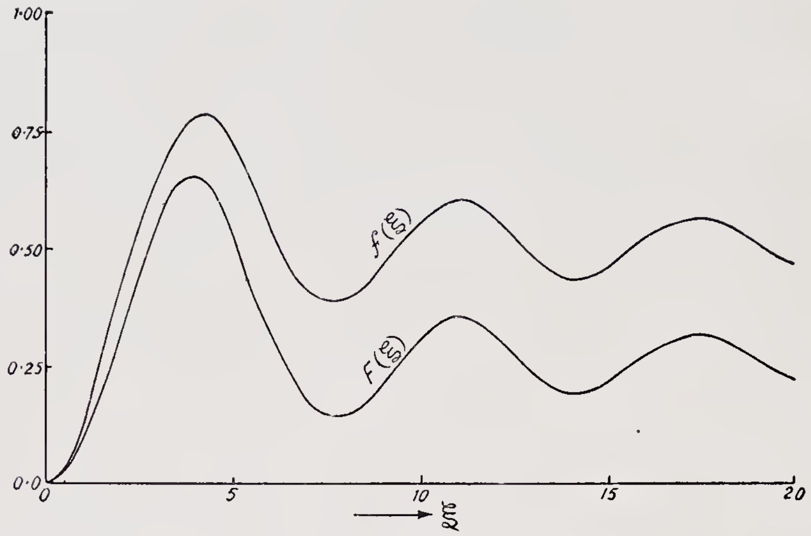


FIG. 68

Graph showing the variation of $F(\xi)$ and $f(\xi)$ with ξ

size of the droplets. The curve starts from the origin, increases in direct proportion to ξ^2, reaches a maximum and then oscillates, finally tending to a value $\frac{1}{4}$. For very small particles, the intensity reduces to the expression

$$\frac{64\pi^4}{9d^2}(\mu-1)^2\frac{a^6}{\lambda^4}.$$

This is identical with Rayleigh's well-known formula for the blue of the sky, except that in our formula we have a factor $4(\mu-1)^2$ instead of $(\mu^2-1)^2$ to which it is nearly equal if μ does not differ much from unity as is assumed in the theory. Thus, in the initial stages, for very small droplets, the theory predicts a preferential scattering of the smallest wavelengths, which is a readily observable phenomenon. For larger particles, the intensity reaches a maximum and then oscillates. Over this range, we may neglect terms in $F(\xi)$ involving higher powers of $1/\xi$ than the first and write

$$I_f = (4\pi^2a^4/\lambda^2d^2)\ (1/4 - \sin\xi/\xi).$$

It is evident from this that the light would show a cyclic change of colours with increasing particle size. Finally, for very large particles, the expression becomes equal to $(\pi^2\ a^4/\lambda^2\ d^2)$, agreeing with the intensity at the centre of the diffraction pattern due to a circular aperture of the same radius.

It is evident also that the superposition upon the primary waves of the radiations scattered forward by the particles in a thin cloud of thickness dl must result in the alteration of both the amplitude and the phase of the latter in its passage through the layer. The former would depend upon the coefficient of $\sin z$ in the expression for the forward scattering and the latter on the coefficient of $\cos z$. Writing the diminution of the amplitude due to the particles in the layer as kdl, and the retardation as $(n-1)dl$, we deduce that

$$k = 2\pi\,\mathrm{N}a^2\left(\frac{1}{2}+\frac{1}{\xi^2}-\frac{\sin\xi}{\xi}-\frac{\cos\xi}{\xi^2}\right) = 2\pi\,\mathrm{N}a^2 f(\xi)\ \text{(say)}$$

$$n-1 = \mathrm{N}\lambda a^2\left(\frac{\sin\xi}{\xi^2}-\frac{\cos\xi}{\xi}\right),$$

where N is the number of particles per unit volume.

The intensity of the incident beam falls off in its passage through the cloud, and after passing through a length l it may be represented by

$$\mathrm{I}_l = \mathrm{I}_0\, \epsilon^{-2kl}$$

I_0 being the intensity of the incident beam. The attenuation coefficient $2k$ is thus proportional to $f(\xi)$. The course of this function with increasing ξ is also plotted in Fig. 66, and is seen to be similar to that of the forward intensity, giving rise to periodic changes in the colour of the transmitted beam also. When ξ is not small, higher powers of $1/\xi$ may be neglected, and the attenuation coefficient becomes $4\pi\mathrm{N}a^2l(\frac{1}{2}-\sin\xi/\xi)$, finally tending to a value $2\pi\mathrm{N}a^2$ which is the same as if the particles were opaque. *Also, the value of the attenuation coefficient is double what would be given by simple geometric considerations, and this may be explained as due to diffraction which introduces an extra loss of energy.*

Since the attenuation coefficient and the intensity of forward scattering undergo similar variations as ξ increases, it is clear that the colour of the source as seen through the cloud must be complementary to that of the light scattered forward as can be observed, for instance, when the sun is seen through the puffs of steam emitted by a locomotive. However, with thick and long columns of cloud, the scattered light itself will be attenuated, and the phenomena thereby modified.

The refractive index of the cloud also undergoes oscillations, alternately becoming greater and less than unity, as the size of the particle steadily increases. In the limiting case of large particles, it becomes practically unity. This is readily explained by the fact that large droplets transmit little light, and such opaque particles can produce no change in refractive index.

So far, we have been considering only the light coming out in the forward direction. When we turn to the diffraction in other directions, it is found that the integral for the amplitude cannot be evaluated completely; but it can be expressed in the form of a series. The method to be adopted depends upon whether ξ is small or large. In the case where coronas are observed, ξ is sufficiently large, and the evaluation may be done by writing $x = \xi \cos \epsilon$, and by repeatedly integrating by parts with respect to x. We then obtain a series, which, on omitting terms containing higher powers of $1/\xi$ than the first, reduces to

$$\frac{2\pi a^2}{\lambda d}\left[\sin Z\left(\frac{\sin \xi}{\xi} - \frac{J_1(\eta)}{\eta}\right) + \cos Z \frac{\cos \xi}{\xi}\right].$$

The contribution of the drop to the intensity is therefore

$$\frac{4\pi^2 a^4}{\lambda^2 d^2}\left[\frac{J_1^2(\eta)}{\eta^2} - \frac{2 \sin \xi}{\xi} \cdot \frac{J_1(\eta)}{\eta} + \frac{1}{\xi^2}\right].$$

The intensity depends both on ξ and η, and hence on the size of the droplets and on the angle of diffraction. Since the function $\sin \xi/\xi$ oscillates and diminishes progressively as ξ increases, the intensity of the corona would fluctuate as a/λ alters, the fluctuations diminishing in extent as the size of the droplets increases. The coronal disc would, in consequence, exhibit colours which are most vivid with the smallest drops, and go through cycles with their saturation progressively diminishing as the drops become larger. The ratio of $\sin \xi/\xi$ to $J_1(\eta)/\eta$ increases rapidly as we move away from the centre of the corona. Hence, the colours would be more prominent towards the margin of the central disc than at its centre. The presence of the function $J_1(\eta)/\eta$ gives rise to alternate bright and dark rings, which can be observed in monochromatic light, but the positions of these rings would be greatly influenced by the value of sin ξ/ξ and the whole appearance of the corona would be different from that of the diffraction pattern of an opaque circular disc. The cyclic changes in this function sin ξ/ξ also give rise to an alternate contraction and expansion of the ring system. In the limit when ξ is sufficiently large, the corona, or at least the central part of it for which η is not too great, tends to become similar to that given by a set of opaque spheres.

Colours of Mixed Plates.—Very beautiful phenomena are shown by the heterogeneous films known as "mixed plates". Though they differ essentially from the coronæ due to water-droplets discussed in the foregoing pages in their nature and origin, there are, nevertheless, some features common to the two cases which justify their being considered in this lecture: To obtain the "mixed plates", a few drops of egg albumen are spread between two plates of glass about ten centimetres square in size and a centimetre thick. The plates are then separated and put back together a few times and slid over each other with a circular movement. The material is thus worked up into a film of uniform thickness which, when seen under the microscope, appears as a thin layer of liquid enclosing a large number of air-bubbles. These vary in size and are irregularly arranged and often depart considerably from a circular shape, but except in special circumstances, show no bias towards elongation in any particular direction. Gorgeous colours are shown by such films when they are freshly prepared and are not too thick. On being allowed to stand, the albumen in the film begins to dry up and forms hexagonal networks between the two plates. The character of the optical phenomena then completely alters.

The colours of mixed plates may be studied in two distinct ways which are roughly analogous to the Haidinger and Newtonian methods of viewing the interferences of transparent plates. The first method is to prepare a film of uniform thickness between flat plates and to view the source of light through the film with the eye placed close behind it and adjusted for distant vision. The second method is to form a mixed plate between two glass lenses in the manner of Newton's rings and to view the illuminated film with the eye placed behind the plate at a suitable distance. As the effects alter with the angles of incidence and observation, the source of light should in either case be of small angular area. An aperture in a screen backed by a filament lamp or a mercury arc may be employed and a dark field of observation should be provided around it. In the first method of observation, the eye observes the light diffracted by the film simultaneously over a wide range of angles. In the second method, films of different thickness are seen simultaneously at nearly the same angle of diffraction; this angle, of course, may be varied by moving the eye laterally. In either case, the angle of incidence of the light on the film may be varied by tilting the plate with reference to the direction of the source.

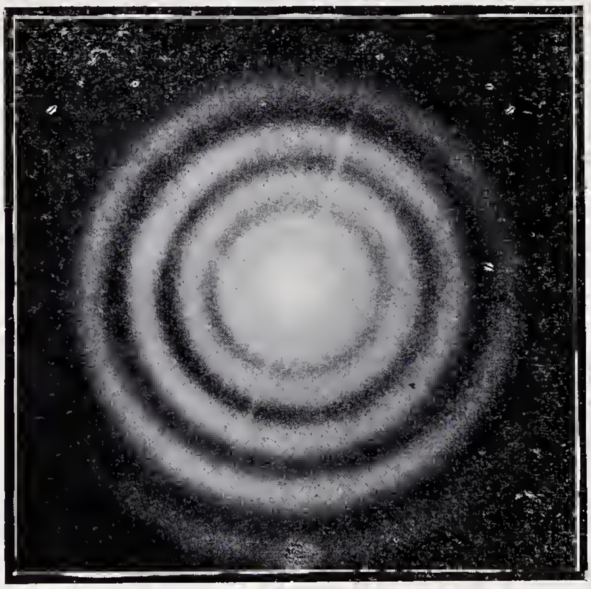


FIG. 69
Diffraction Halo of Mixed Plates

The characteristic effect* of which the explanation largely covers the whole theory of mixed plates is the diffraction halo seen around a bright source of light viewed normally through the film (see Fig. 69). This halo consists, in monochromatic light, of

* C. V. Raman and B. Banerji, *Phil. Mag.*, 1921, 41, 338.

a series of circular rings, alternately bright and dark, which concentrically surround the source. The rings are narrowest nearest the centre of the halo and widen as we proceed towards its outer margin. If white light is used, the outermost ring is practically achromatic and is followed within by coloured rings. A thick plate shows numerous close rings, while a thin plate shows fewer rings which are wide apart. The rings move inwards when the thickness of the film is reduced. Thus, the thinner the plate, the more striking are the colours shown by the rings nearest the centre of the halo. A source of white light viewed through the film appears dimmed in intensity and exhibits a hue complementary to the colour of the part of the halo actually overlying it. A monochromatic light source fluctuates in intensity when the thickness of the films through which it is viewed is altered, appearing brightest when the halo has a dark ring at the centre, and feeblest when the source is overlaid by a bright ring. The rings near the centre of the halo show peculiar variations in their visibility depending on the thickness of the plate through which the source is viewed, sometimes being scarcely observable and sometimes very vivid and clear. Such fluctuations are not shown by the outer rings in the halo. The observations indicate that there is a second ring-system of small angular extension superimposed upon the main system and affecting its visibility when the two sets of rings are not in coincidence in any particular direction.

It is clear from the facts already stated, that the character of the halo is determined by the thickness of the liquid-air film and not by the size or shape of the air bubbles in it. It is also evident that the halo registers the characteristics of the diffracted radiation from the laminar edges in the film.* Each line element of the edge diffracts light principally in a plane normal to its own direction; the part which proceeds towards the air-side of the boundary may be referred to as exterior diffraction, and the parts towards the liquid side as interior diffraction. The existence of both types of diffraction in equal intensity but with opposite phases at small angles with the incident beam is shown by the

* C. V. Raman and B. Banerji, *Phil. Mag.*, 1921, 41, 860.

Foucalt test. As in the case of the striæ in mica, the laminar boundaries in mixed plates appear as brilliantly coloured *double lines* when the light is blocked out at the focus, the colour being complementary to that of the central fringe in the Fresnel diffraction patterns (Fig. 70).

FIG. 70
Mixed Plates in the Foucalt Test

Though the interior and exterior diffractions by the laminar boundary are symmetric *at small angles,* they cease to be so at larger angles. The interior diffraction is much more intense and is visible over a wide range of angles, whereas the exterior diffraction rapidly diminishes in intensity and vanishes when the angle of diffraction exceeds a few degrees. This is readily seen on illuminating the film and viewing it obliquely through a microscope. The two halves of the edge of each bubble appear of very different intensities and indeed one half very quickly vanishes, while the other half remains visible but shortens into a crescent as the obliquity is increased.* The reason for these facts is obvious

* I. R. Rao, *Ind. Jour. Phys.*, 1927, 2, 167. Some photographs illustrating the effect are reproduced with this paper.

when we consider the form of the laminar boundaries which owing to the action of surface tension have a specific shape independent of the size of the bubble, namely, a semi-circular arc whose diameter is equal to the thickness of the film. The manner in which the wave-front of the light is modified in its passage through the film is indicated in Fig. 71. The wave incident on the

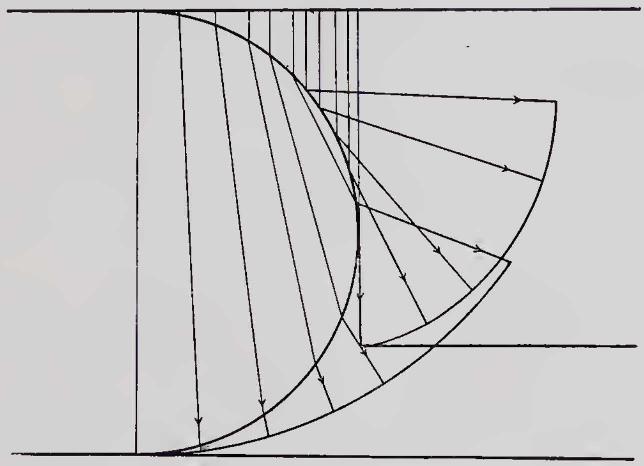


FIG. 71
Form of Wave-Front in Mixed Plates

curved liquid-air boundary is in part twice refracted and in part totally reflected at the interface. The twice-refracted part forms a curved continuation of the wave-front which has passed through the air, while the totally reflected part forms a cusp-like appendage to the wave-front which has passed through the liquid. The appearance of the diffracted radiation towards the interior is thus strongly favoured, while towards the exterior it is greatly weakened. The explanation of the asymmetry of diffraction indicated by Fig. 71 is completely confirmed by viewing the edge under sufficiently high powers of the microscope. The emergence of the refracted and reflected rays from distinct points on the meniscus and their approach to each other with increasing obliquity can actually be observed when the films are fairly thick. The disappearance of the exterior diffraction at larger angles is found to occur more rapidly with films of greater thickness.

It is thus evident that the interior and exterior diffractions by the laminar boundaries appear superposed in the halo. The two are of the same intensity at small angles of diffraction, but at larger angles the interior diffraction is much more intense and principally determines the observed phenomena. In either case, the edge radiations are derived simultaneously from the two parts of the wave-front which have passed through the film. If we ignore the light transmitted or reflected by the liquid by the liquid meniscus, the edge radiations from the wave-fronts which emerge from liquid and air respectively would interfere under a path difference

$$(\mu - 1)\ d - \tfrac{1}{2}\ d \cdot \sin\ \theta - \tfrac{1}{2}\ \lambda$$

for interior diffraction, and under a path difference

$$(\mu - 1)\ d + \tfrac{1}{2}\ d\ \sin\ \theta + \tfrac{1}{2}\ \lambda$$

for exterior diffraction, θ being the angle of diffraction, μ the refractive index of the liquid and d is the diameter of the meniscus, which is also the thickness of the film. When θ is sufficiently small, $\frac{1}{2} d \sin \theta$ may be neglected and the expressions show that the colour of the diffracted light would be complementary to the interference colour of the light transmitted through the film. For larger angles of diffraction, the path difference increases for exterior diffraction and diminishes for interior diffraction. But at such angles, the effects due to the meniscus become of great importance in interior diffraction. Here the case may be treated as practically one of interference between the rays which are totally reflected and those twice refracted at the meniscus. Their path difference is easily shown to be

$$d\ (1 - \mu \cdot \sin\ i)\ (\mu\ \cos\ i - \sqrt{1 - \mu^2\ \sin^2\ i}) - \delta,$$

where i is the angle of incidence at the meniscus of the light which is twice refracted, and δ is the correction for the change of phase in total reflection. The angle of diffraction θ of the light emerging from the film is given by the formula

$$\sin\ \theta = \mu\ \sin\ 2\ (r - i), \text{ where } \sin\ i = \mu\ \sin\ r.$$

For small values of i and θ, it is readily shown that the path difference given by this formula is sensibly the same as in the one given above, namely,

$$a\ (\mu - 1) - \tfrac{1}{2}\ d\ \sin\ \theta - \tfrac{1}{2}\ \lambda.$$

For larger values of i and θ, the path difference falls off more rapidly, finally vanishing when i is equal to the critical angle for the liquid and $\sin \theta = \mu \sin 2i$. The corresponding direction of emergence of the light from the film would be outside the observable limit of the diffraction halo.

The diffraction halo as observed thus consists of two sets of rings, the intensities of which in any direction are superposed. In one of them, the path difference of the interfering rays diminishes with increasing angle of diffraction and finally vanishes in the direction of the achromatic ring. In the other set of rings which has a relatively small angular extension, the path difference becomes larger with the increasing angle of diffraction. The superposition of the two sets of rings whose angular positions are not the same thus leads to fluctuations in their visibility at small angles. From the formulæ, the angular positions of the rings due to interior diffraction can be calculated and compared with observation and a satisfactory agreement is found.* The formula also enables a calculation to be made of the diameters of the dark and bright rings localised on a film of non-uniform thickness at any given angle of observation, and the particular angle at which a blurring of the rings would occur for a given thickness of the film. In every case the theory is confirmed by the actual measurements. Since the phase change occurring in total reflection is different for light polarised in and at right angles to the plane of incidence, there should be a corresponding small difference in the positions of the rings in the two cases. Even this fine point in the theory is confirmed by observation.† It is noticed that a plate which is too thick to show colours when viewed normally shows them if seen obliquely. Further, a film which shows colours when viewed normally appears achromatic when observed obliquely. These facts receive a satisfactory explanation on the theory.

It may be remarked that the edge of each bubble in the film gives the complete diffraction halo, the diameter of the rings

* C. V. Raman and K. Seshagiri Rao, *Phil. Mag.*, 1921, 42, 679.

† It follows that if the incident light be plane-polarised in an arbitrary azimuth, the light diffracted at the boundary would, in general, be elliptically polarised.

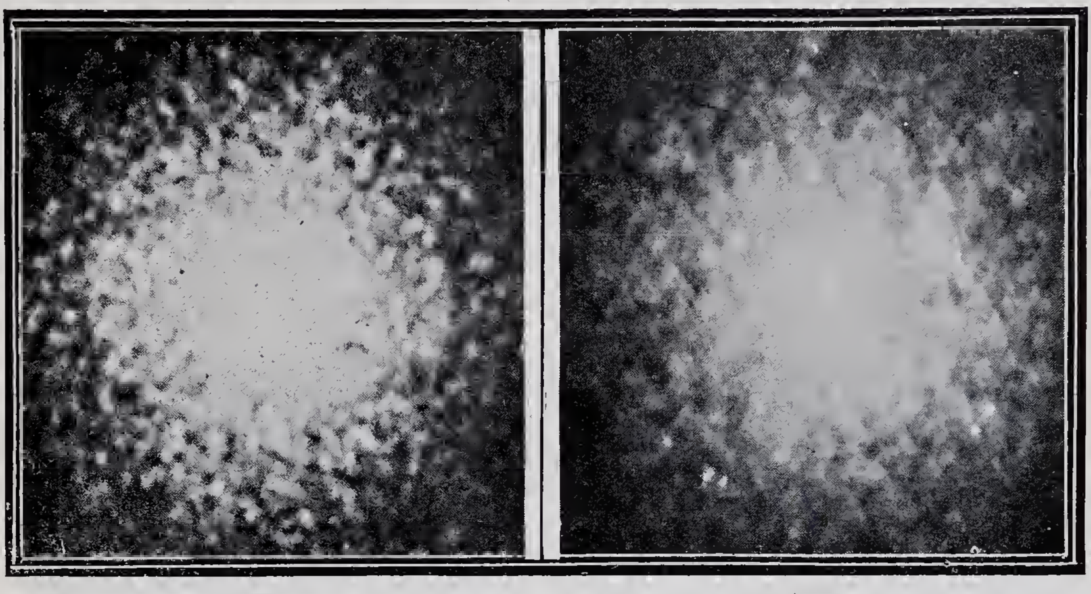
being, however, independent of the size of the bubble. The intensity of the halo in any particular direction depends on the aggregate length of the laminar edges running in the perpendicular direction. Hence, if the bubbles show a bias towards elongation in any particular direction, the halo appears intensified in the transverse direction, the rings, however, remaining circular.

An easy extension of the theory enables the oval haloes observed with obliquely held plates and the corresponding phenomena with non-uniform plates to be explained. As already mentioned, dry films exhibit phenomena of a quite different nature. For these and other details, reference may be made to the original papers.*

Intensity Fluctuations in Coronæ.—We shall now proceed to a closer examination of the nature of the diffraction pattern produced by a randomly distributed cloud of particles. As remarked earlier, such a pattern is *statistically* a summation of the effects of the individual particles but differs from them vastly in detail. Fig. 66 (*a*) on page 137 exhibits the central region of the corona observed around a *monochromatic* source of light of small angular extension, when viewed through a glass plate lightly dusted with lycopodium. The central disk of the corona is over-exposed in the photograph and shows no detail, but the granular structure of the pattern is seen very clearly in the first ring surrounding it. *Each of the bright spots in the field is a focussed image of the original source of light, formed by the joint action of the diffracting particles and the lens of the photographic camera.* This is verified by varying the size or shape of the source of light and noting its effect on the appearance of the pattern. It is then noticed that all the bright spots in the field alter in the same way and have the same form as the source. This is illustrated in Fig. 72 which shows the central disc of the corona photographed with a smaller exposure and on a larger scale than in Fig. 66 (*a*), so as to clearly bring out the structure of the pattern. A small circular aperture and another in the form of a somewhat larger equilateral triangle were used as sources in photographing the

* See also K. Seshagiri Rao, *Proc. Ind. Assoc. Cult. Sci.*, 1923, **8**, 243. In this paper, the intensity distribution in the diffraction halo of mixed plates and the phenomena presented by dry films are discussed.

two patterns reproduced. The circular and triangular shapes of the individual spots appearing in Figs. 72 (*a*) and (*b*) can easily be recognised. The triangles in Fig. 72 (*b*) appear inverted on the plate with respect to the source, as they should be in the images formed by a converging lens.*



(*a*) (*b*)

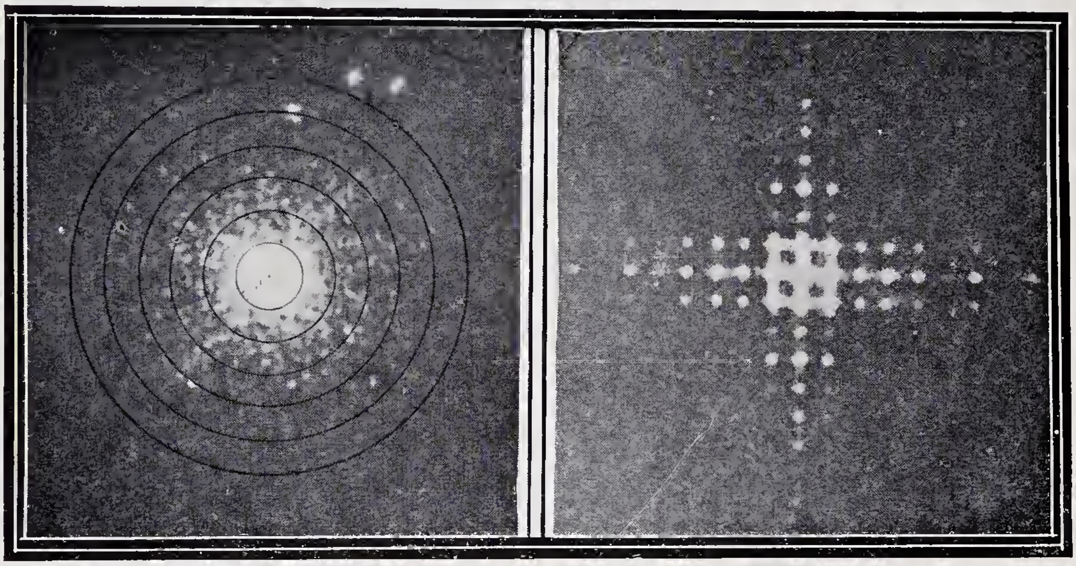
Fig. 72

Central Disc of Corona in Monochromatic Light with (*a*) a Circular Pin-hole and (*b*) a Triangular Aperture as Source

It is familiar knowledge that a regularly spaced arrangement of apertures or obstacles can function as a diffraction grating and in combination with a lens give focussed spectra which in effect are monochromatic images of the source of light employed. The patterns reproduced in Fig. 72 show that a perfectly random arrangement of diffracting apertures or obstacles can also give well-defined images; the superiority of the regular grating is that it gives fewer and correspondingly more intense images in easily calculable positions instead of a great many feeble and irregularly spaced ones. The results are readily understood, since the optical effect in the focal plane of the lens can always be regarded as due to a plane wave of appropriate amplitude covering the entire area of the lens and travelling in such a direction that it

* G. N. Ramachandran, *Proc. Ind. Acad. Sci.*, A, 1943, 18, 190.

comes to a focus at the point under consideration. The definition of the image of the source appearing at such point would be determined in every case by the configuration of the boundary of the lens and not by the disposition of the individual apertures or obstacles over its area. That the images formed by a random distribution of diffracting particles are not inferior in definition to those given by a regular diffraction grating is illustrated in Figs. 73 (*a*) and (*b*). These reproduce respectively the central part of the corona observed through a glass plate dusted with lycopodium and the diffraction spectra given by a fine sieve of metallic wires. A fine pin-hole illuminated by the 5461 Å radiation of a mercury lamp was the source and the optical conditions were also otherwise completely identical in the two cases.



(*a*) (*b*)

Fig. 73

Comparison of Corona with Diffraction Spectra given by a Grating: (*a*) Corona and (*b*) Diffraction Spectra

The relation between the structure of the corona and the distribution of the diffracting particles on the plate can be illustrated in various ways. If, for example, the plate is moved in front of the eye, keeping the latter fixed on the source, the ring-system does not undergo any change, but the fine structure of the corona appears to move relative to the pattern of rings in *the same direction* as the motion of the plate. *Vice versa,* if one moves the eye, keeping the plate fixed, all the while looking at the source, the structure of the corona appears to move in *the opposite direction.* If the plate is rotated, the structure rotates in the same direction. The prettiest effects are those observed when a very small aperture is held immediately before the eye so as to limit the effective area of the lycopodium-dusted plate held in front of it. As the plate is moved relative to the aperture, different areas of the former become operative, and the spots in the corona appear and vanish at random positions in the field, thus simulating the effects seen in a spinthariscope.

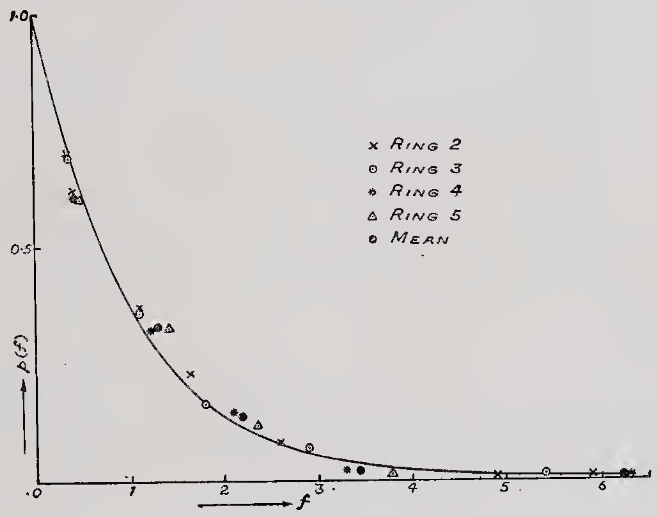


FIG. 74

Verification of the Statistical Law of Intensity Fluctuation

The theoretical law of distribution of intensities resulting from random interferences which was derived earlier, *viz.*, that $dW = p(f)df = \exp(-f)df$ has been tested* making use of the photograph reproduced in Fig. 73 (a) for counting the spots and classifying them according to their observed intensities. The average intensity in a corona falls away from the centre in the proportion $J_1^2(x)/x^2$, where $x = 2\pi\ a \sin \phi/\lambda$, a being the radius of the particles, ϕ the angle of diffraction, and λ the wavelength of the light. To take account of this factor, the parts of the corona where the spots could be clearly seen were divided into five annular regions as marked in the photograph; the spots in each

* G. N. Ramachandran, *Proc. Ind. Acad. Sci.*, A, 1943, 18, 190.

annulus were counted and classified in a scale of intensities established with the aid of the diffraction pattern of the grating photographed under strictly comparable conditions [Fig. 73(b)]. Using this tabulated list, the average intensity for each region was computed, and thus the values of $p(f)$ and f were determined for that region. These were plotted in Fig. 74 for four regions (the innermost one being too dense to facilitate counting), the continuous curve in the figure being the one calculated from theory. A final average for the four regions was obtained by dividing the intensities of the spots in each by the mean value of $J_1^2(x)/x^2$ for it, and the values of $p(f)$ thus obtained appear represented by black dots in Fig. 74. It will be seen from this figure that the experimental values fit the exponential formula indicated by the theory remarkably well.

The Phenomenon of the Radiant Spectrum.—Since, as we have seen, the bright spots in a diffraction corona as seen with monochromatic light are real images of the source employed, it follows that when white light is used instead, each bright spot would be drawn out into a spectrum, the different radiations appearing at distances from the centre of the pattern proportional to their respective wave-lengths. This explains why in such circumstances, coronæ exhibit numerous long coloured streamers or spectra located at random but directed radially outwards from the centre of the pattern. The streamers are most clearly seen in the outer parts of the corona, traversing its marginal rings and extending to the farthest visible limits of its extension. The streamers are distinguishable also in the central disc of the corona [see Fig. 66(b) on page 137], but their radial distribution and their colours are least conspicuous near the centre of the pattern. It is interesting to observe the coronæ through a filter which transmits only two well-separated regions in the spectrum, *e.g.*, the red and green regions. The entire pattern then appears filled with red and green spots; every green spot is accompanied by a red one, the two being along the same radius and the red spot at a distance from the centre greater than that of the green spot in the proportion of the two wave-lengths. It is also of interest to view the diffraction corona through a dispersing prism held in front of the eye. The source of light then itself appears drawn out into a spectrum, and the radiant spectra are drawn out or shortened and also tilted one way or another, according to the direction in which they run. As the result of these changes, the "achromatic centre" of the diffraction pattern from which the coloured streaks appear to diverge is shifted away from its original position to a point lying well beyond the violet end of the spectrum into which the light-source is itself seen dispersed.

A familiar example of "radiant spectra" are those noticed when a small intense source of light is viewed directly by the normal eye against a dark background. Long coloured streamers of light are seen to diverge from the source in all directions, and faint coloured haloes also appear encircling the source near the outer limit of the streamers. Curious movements are also noticed within these streamers which may be controlled to some extent by fixing the eye on the source. The fact that the streamers disappear and are replaced by numerous bright points of illumination when a monochromatic source is used instead of white light, clearly indicates that we are here dealing with diffraction effects analogous to those discussed in the foregoing pages.* The diffracting structures are evidently those present in the refractive media of the eye, including especially the cornea, and the crystalline lens, and possibly also the vitreous and aqueous humors. To give rise to such effects, it is not necessary that the diffracting particles should be opaque or spherical or of uniform size. Even small differences of refractive index in regions of appropriate size should be sufficient to give the observed phenomena. The angular dimensions of the brightest region of the diffraction corona are in accord with the supposition that it owes its origin to the known structure of the cornea of the eye, while it appears probable that the outer coloured haloes arise from the fibrous structure of the crystalline lens around its margin.

Holding a dispersing prism in front of the eye and viewing a bright source of light through it, the radiant streamers now appear to diverge from a point well beyond the violet end of the spectrum into which the light-source is itself dispersed. This

* C. V. Raman, *Phil. Mag.*, 1919, 38, 568.

effect, noticed long ago by Brewster, is clearly analogous to that observed with diffraction coronæ and discussed above.* It is, of course, necessary that the prism used should have clean and well-polished surfaces so that it does not itself give rise to disturbing effects of a similar nature.‡

Observation of Brownian Movements without a Microscope.—As illustrated by Figs. 72 (a) and 73 (a) appearing on earlier pages, the corona due to a cloud of diffracting particles exhibits strongly marked local variations of intensity. These variations are determined by the distribution of the diffracting particles in space, and if this alters with time, there would necessarily be corresponding changes in the corona. If the movements of the particles are large and rapid, all trace of visible structure would disappear from the field. If, however, the movements are sufficiently small and slow, it should be possible to follow the changes in the corona from instant to instant and thus obtain visual evidence that the diffracting particles are in motion.

As is well known, the individual particles in colloidal suspensions and emulsions execute "Brownian movements", which are most lively when the particles are very small and are suspended in an inviscid fluid. For our present purpose, it is necessary to select a substance in which the particles are of fair size so that the coronal disc is of sufficient intensity and also exhibits a visible structure. Fresh milk is the most easily available material satisfying this requirement. When a little of it is flowed on to a clean glass plate and then allowed to drain away as completely as possible, a thin film remains firmly adherent to the plate. A small aperture illuminated by a mercury arc lamp and viewed through such a film exhibits an extended field of diffuse illumination surrounding it. Fixing the attention over a limited area of the field, it is noticed that this exhibits a structure which is not static but is continually changing. Bright points of illumination continually appear in the field and others disappear. These changes become less rapid and ultimately stop when the film is dry; the structure of the field is then completely static.§

* C. V. Raman, *Phil. Mag.*, 1922, 43, 357.
‡ C. V. Raman, *Nature*, 1922, 109, 175.
§ Unpublished observations by the author and G. N. Ramachandran.